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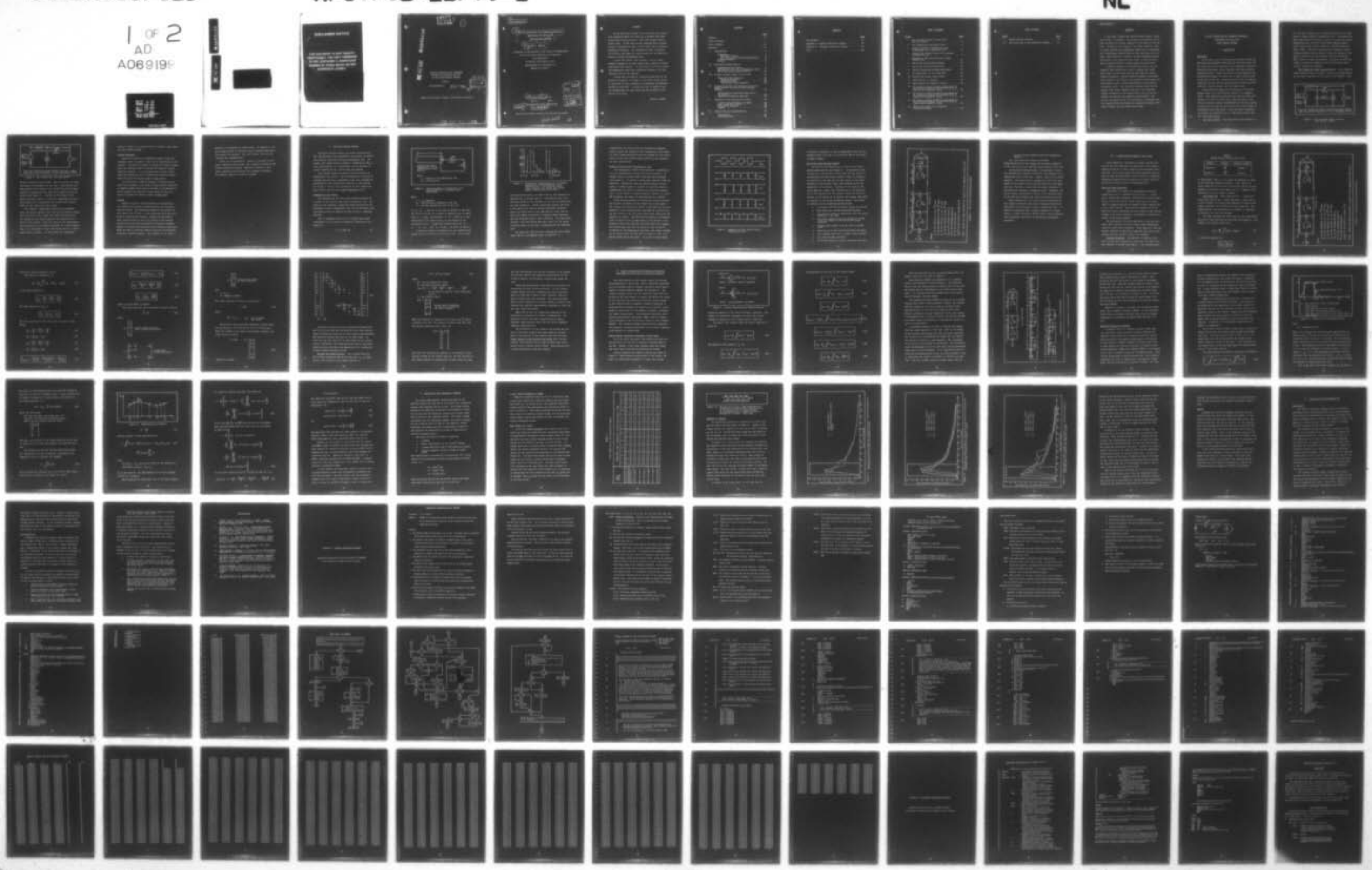

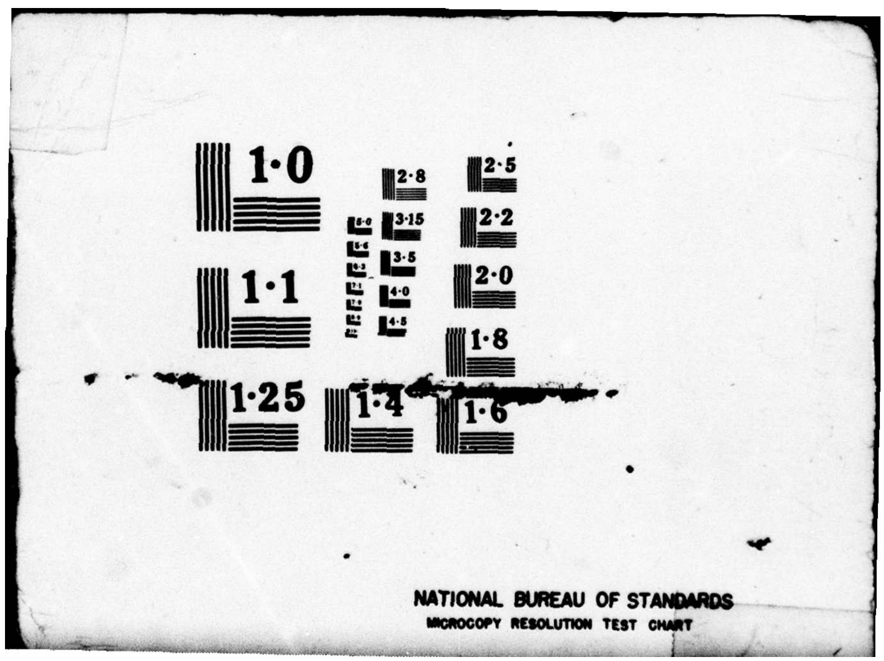
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DIGITAL SIMULATION AND PARAMETER
ESTIMATION TECHNIQUES FOR THE
E-LINE PULSE-FORMING NETWORK

THESIS

AFIT/GE/EE/79-2

Larry W. Vannoy
2nd Lt USAF

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DIGITAL SIMULATION AND PARAMETER ESTIMATION TECHNIQUES FOR THE E-LINE PULSE-FORMING NETWORK.

9 Master's THESIS

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by
10 Larry W. Vannoy B.S.
2nd Lt USAF
Graduate Electrical Engineering

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<u>Preface</u>

In the High Power Branch of the Air Force Aero Propulsion Laboratory there is a need for a reliable and lightweight pulse-forming network. This need brought about a thesis topic. In the topic it was proposed that a student develop a mathematical model of the network and a parameter estimation technique. The technique is used to estimate the inductor and capacitor values needed for the network to produce a desired rectangular pulse.

I chose this thesis topic because I have an undergraduate background in power systems and a desire to better understand these systems. In this thesis I not only develop the mathematical model and estimation technique, but also implement them on the computer.

I would like to thank Dr. Fredrick Brockhurst for his guidance throughout all stages of the thesis effort and Dr. J. Gary Reid for his assistance in developing the parameter estimation algorithm. I would also like to thank my wife for transcribing the rough draft and her tolerance of me during the writing.

Larry W. Vannoy

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Abstract

A line-type, voltage-fed, pulse-forming network, called the "E-line," is studied. The energy to be delivered to the load is stored in a source capacitor at the front of the network. Storing all the energy in the source capacitor leads to a more reliable and lightweight system than is achievable by a conventional network where all the capacitors store energy. The network is to deliver 100-200 rectangular pulses per second to the load where each pulse is 20-30 microseconds in duration and has a pulse height of 30,000 volts.

A computer program is developed which models the discharge of the network and is used to analyze the output pulse shape for different inductor and capacitor values within the network. Another computer program is developed which estimates the inductor and capacitor values needed for the network to give a close approximation to the desired rectangular pulse. The simulation program models the discharge of the actual network very well. The estimation program as applied in this thesis does a poor job of predicting the inductor and capacitor values needed. The desired rectangular pulse is not achieved. Recommendations are included which may improve the simulation program, estimation program, and the actual pulse shape.

DIGITAL SIMULATION AND PARAMETER ESTIMATION TECHNIQUES FOR THE E-LINE PULSE-FORMING NETWORK

I. Introduction

Background

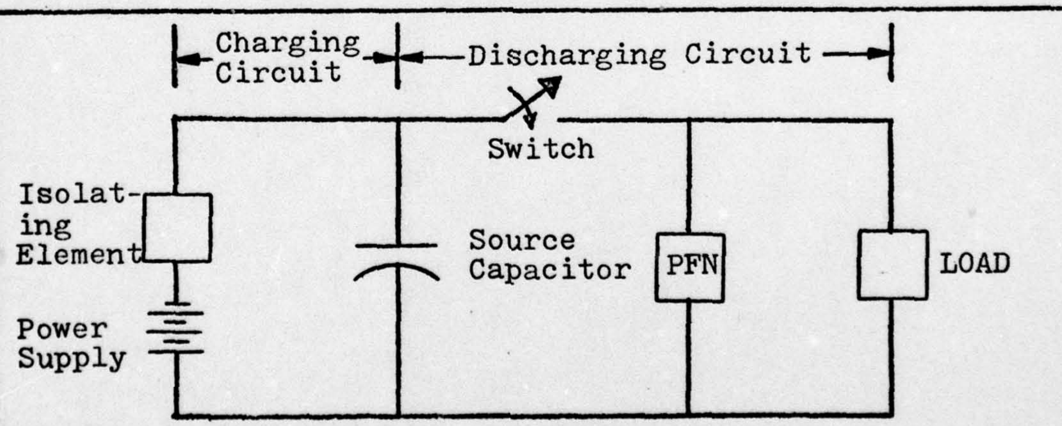
In the 1940's one of the technical problems facing the scientists and engineers developing the microwave radar for the armed services was devising a pulse generator capable of delivering high power, short duration, and high recurrence frequency pulses. Today, the number of pulse generator applications has grown, but their use in radar systems stays at the top of the list. Along with some of the new applications has come the need for pulse generators which are more efficient, lighter weight, and have higher power.

Pulse generators depend on the storage of electrical energy either in an electrostatic field or in a magnetic field. Pulse generators logically fall into two major categories, (1) those in which only a small fraction of the stored electrical energy is discharged into the load during a pulse, and (2) those in which all of the stored energy is discharged during each pulse. These two basic categories of pulsers are referred to as (1) "hard-tube pulsers" and (2) "line-type pulsers."

Line-Type Pulsers. Line-type pulsers are referred to

as "line-type" because their energy storage device is essentially a lumped-constant transmission line. The energy storage element serves not only as a source of electrical energy during the pulse, but also as the pulse shaping element, thus it has become commonly known as a "pulse-forming network," (PFN). There are essentially two classes of pulse-forming networks, namely, those in which the energy for the pulse is stored in an electrostatic field in the amount, $\frac{1}{2}CV^2$, called "voltage-fed networks," and those in which the energy is stored in a magnetic field in the amount, $\frac{1}{2}LI^2$, called "current-fed networks."

<u>The Voltage-Fed Pulse-Forming Network</u>. The voltage-fed pulse-forming network in Figure 1 has all its energy stored in the source capacitor at the front of the network. This



where the isolating element isolates the power supply from the current transients of the discharging circuit

Figure 1. The line-type pulser on which this thesis is based.

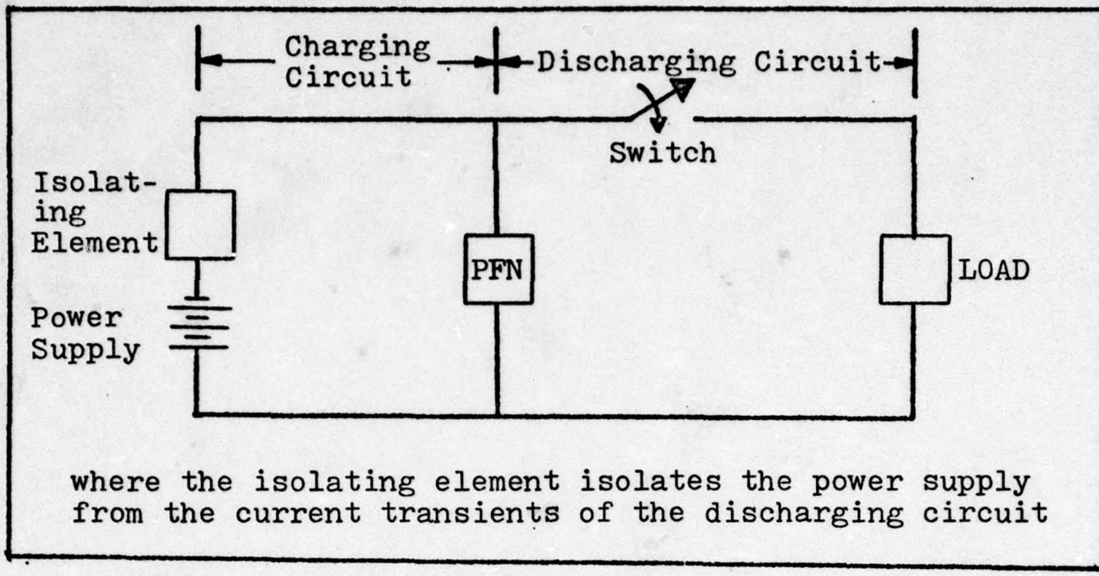


Figure 2. The conventional line-type pulser.

thesis is based on this network. When the switch is closed, the energy is discharged into the load. Conventional line-type pulsers store energy in the voltage-fed pulse-forming network by charging all the capacitive elements within the network (see Figure 2). Storing all the energy in the source capacitor leads to a much more lightweight and reliable system than is achievable by a conventional network where all the capacitors store energy.

The reliable, lightweight system has one drawback; since the source capacitor must hold all the network energy, its value must be much larger than would be necessary if the energy had been shared among all the capacitive elements of the network. Since the values of the capacitances and inductances of the network determine the energy transfer time, it is desirable to keep these values small if a short pulse width is desired. In this thesis the lightweight

system of Figure 1 is explored with an attempt to get around the pulse width problem.

Problem Statement

There is a need for a mathematical model of the discharging circuit in Figure 1 which can be implemented on the computer. The model is needed to analyze the output voltage pulse for different sets of inductor and capacitor values within the network. It is also desirable to have a technique for finding the set of inductor and capacitor values which will give the desired output voltage pulse.

In this thesis a computer program is presented which models the discharge of the pulse-forming network along with a computer program which makes an attempt at estimating the set of inductor and capacitor values which will give as close as possible the desired output voltage pulse.

Outline

Chapter II gives a brief discussion on transmission line theory and how pulse-forming networks were developed from the theory. The particular network to be explored in this thesis, the "E-line," is also presented with specifications to be met by the network. Chapter III develops a state variable model of the E-line needed in the network simulation and the inductor-capacitor estimation programs. Chapter IV presents the simulation and estimation algorithms which are used to form the network simulation and parameter estimation computer programs. The programs can be found in

Appendix A and Appendix B, respectively. In Chapter V, output voltage pulses for different sets of inductor and capacitor values are analyzed. The final chapter contains conclusions and recommendations.

There are two appendices. Appendix A contains a flowchart, operating instructions, and a computer listing of the network simulation program. Appendix B contains a flowchart, operating instructions, and a computer listing of the inductor-capacitor estimation program.

II. The Pulse-Forming Network

The pulse-forming network, here after referred to as PFN, was developed from transmission line theory, in particular from the open-ended lossless transmission line. Although a PFN cannot exactly simulate a transmission line, a close approximation to a transmission line is possible if circuit elements of the correct value are present.

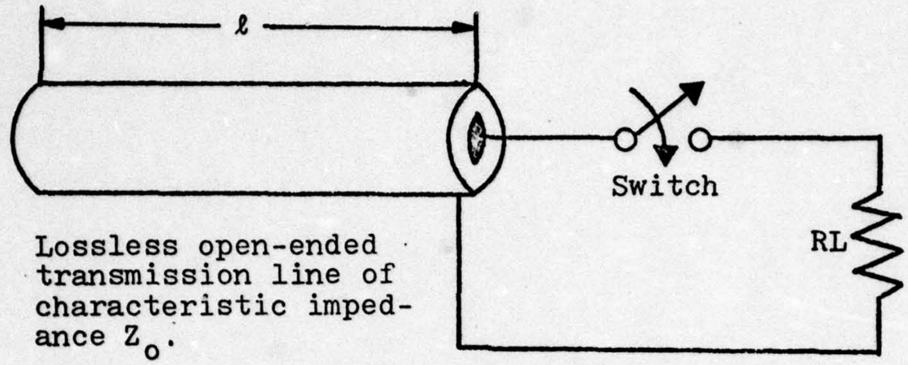
This chapter begins by giving a brief discussion on transmission line theory. This is followed by a section on networks derived from a transmission line, and concludes by discussing the PFN of particular interest in this thesis, the "E-line." Much of the information in this chapter has been derived from the book, Pulse Generators (Ref 7).

Transmission Line Theory

The transmission line that pulse-forming networks try to model is the lossless open-ended transmission line. The nature of the transient voltage and current produced by the discharge of such a transmission line into a resistance load (see Figure 3) may be studied in terms of its a-c impedance function.

The a-c impedance function of a lossless open-ended transmission line is given by elementary transmission line theory as:

$$Z = Z_o \coth j\omega\delta \tag{1}$$



Lossless open-ended transmission line of characteristic impedance Z_o.

where:

ℓ = length of the transmission line

RL = load resistor

Figure 3. Circuit diagram of transmission line discharging into a resistive load (Ref 7:177).

where

Z = a-c impedance

Z_o = characteristic impedance of the line

δ = one way transmission time of the line

If $R_\ell = Z_o$, that is, if the line is matched to the load, the current consists of a single rectangular pulse of amplitude $I_\ell = V_o/2Z_o$ and duration $\tau = 2\delta$, where I_ℓ is the transmission line current and V_o is the voltage on the transmission line. Voltage and current pulses for $R_\ell = Z_o$, $R_\ell = 2Z_o$, and $R_\ell = (1/2)Z_o$ are shown in Figure 4.

A series of steps is introduced into the transient discharge when the line does not match the load. These steps

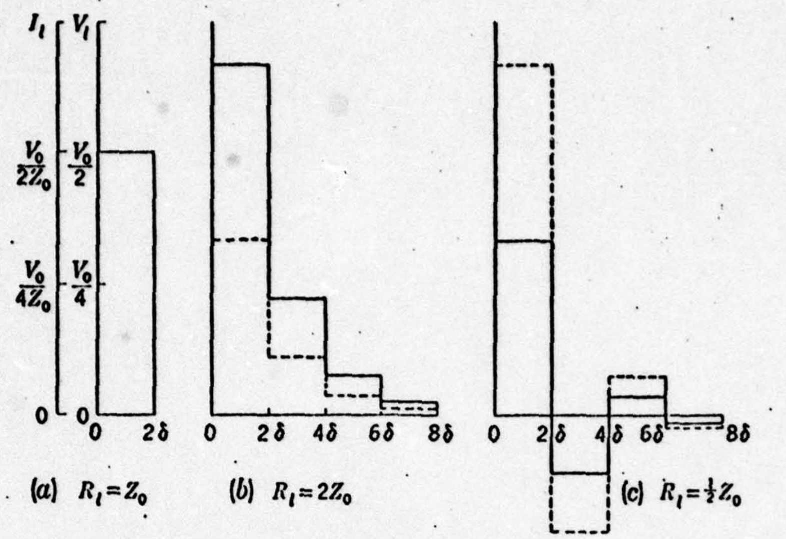


Figure 4. Current and voltage pulses for a lossless transmission line discharging into a resistive load. The solid and broken lines represent the voltage and current pulses, respectively (Ref 7:178).

are all positive when R_ℓ is larger than Z_o and alternate in sign when R_ℓ is smaller than Z_o. The steps are due to reflections caused at the terminals of the line by mismatching the load resistance. "These reflections traverse the line to the open end in time δ, and are completely reflected there, and travel back to the load end in a total time 2δ, where they appear as positive or negative steps depending upon the mismatched ratio" (Ref 7:178-179). The reflections continue, with diminishing amplitude, until all the energy initially stored in the line is dissipated in the resistive load.

One might ask, "Why not use a transmission line rather than a PFN if a rectangular pulse is desired?" A

transmission line is not used for producing rectangular pulses because the diameter of the transmission line needed for high voltage applications and the length the line would have to be for the correct pulse duration is not practical for most applications.

Networks Derived From a Transmission Line

The development of PFN's which simulate a transmission line is a mathematical problem in network synthesis. At present there are several PFN's that have been developed (see Figure 5). It should be noted that, although a PFN can simulate a transmission line, no network having a finite number of lumped parameter elements can exactly simulate a transmission line which in reality has distributed parameters. As the number of lumped parameter elements in a given PFN increases, the degree of approximation will improve.

Pulse forming networks are essentially ladder networks. The sections or windows of the network are called meshes. The voltage drop across and current in any mesh are not only dependent on the elements within the mesh but also the voltage and current in all the other meshes taken separately and together as a group. The PFN will not produce a rectangular pulse if there are not enough meshes in the network or the elements within each mesh have the wrong value; for example, the network pulse may rise too slowly or have excessive overshoots at the beginning of the pulse, may ripple throughout the pulse width, and may fall too slowly or have overshoots at the end of the pulse. It is part of this thesis

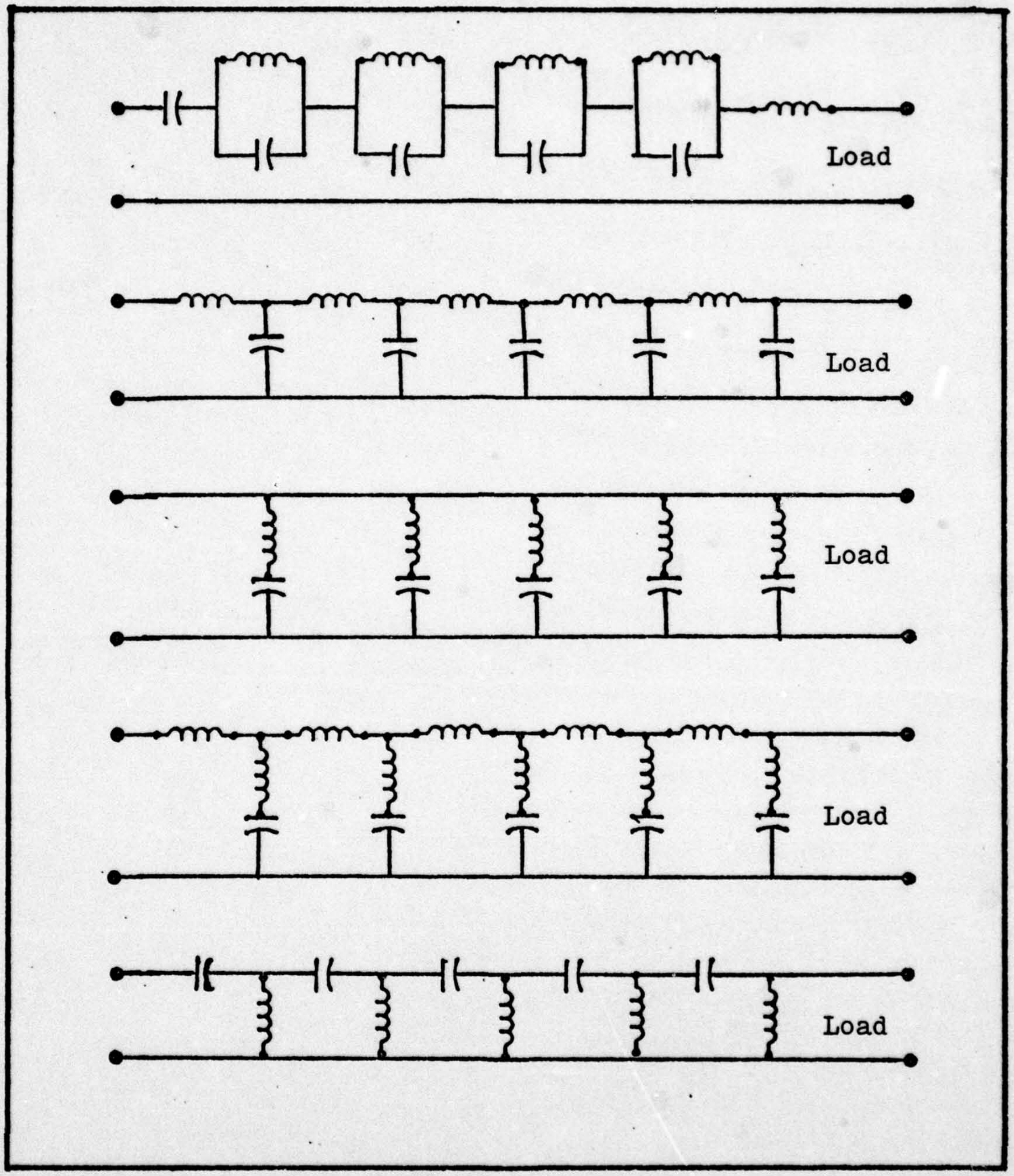


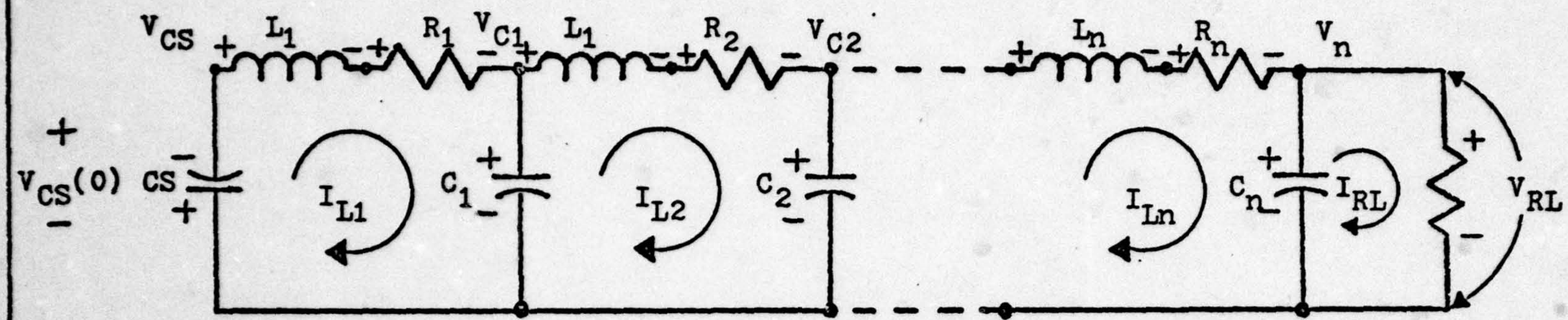
Figure 5. Examples of PFN's derived from a Transmission Line.

to develop a technique to find an approximate value for the elements within each mesh of a particular PFN as the number of meshes changes.

The E-Line Pulse-Forming Network

The rest of this thesis will be based on the E-line pulse-forming network (see Figure 6). As mentioned in the introduction, this network differs from the type used by a conventional line-type pulser scheme in that the energy to be delivered to the load has all been stored in the first capacitor of the network rather than charging up all the capacitors of the network. Storing all the energy in the first capacitor, rather than storing some of the energy in all of the capacitors of the network, causes the first capacitor of the network to be several times larger than would be required if the energy storage was shared. The E-line of interest has the following specifications:

1. It produces 100-200 rectangular pulses per second.
2. Each pulse has a 20-30 microsecond duration.
3. All possible energy is transferred from the source capacitor to the load.
4. The source capacitor has been charged to 30,000 volts and contains 60,000 joules of energy initially.
5. Voltage pulse height is on the order of 30,000 volts.
6. There are to be few oscillations after the pulse.
7. The charging capacitor is charged between pulses.
8. The load is purely resistive.
9. At initial start up, initial conditions are zero



$i=1,2,3,\cdots n$

where:

L_i is the inductor in the i^{th} mesh

C_i is the capacitor in the i^{th} mesh

R_i is the resistor in the i^{th} mesh

I_{Li} is the current in the i^{th} mesh

V_{Ci} is the voltage across C_i

CS is the charging capacitor

RL is the load resistor

$V_{CS}(0)$ is the initial voltage on CS

Note: notice the sign convention on $V_{CS}(0)$ is the opposite of that on CS

Figure 6. The E-Line Pulse-Forming Network

on all circuit elements except the charging capacitor.

10. The E-line will contain 5-7 meshes.

Transmitting most of the energy from the source capacitor to the load is the most important specification in the list.

In any pulse-forming network it is desirable to have the characteristic impedance match the load impedance so no reflections occur in the line. However, this is not always possible. The characteristic impedance of the E-line analyzed in this thesis in fact does not match the load impedance and, therefore, reflections add another dimension to the problem of getting a rectangular pulse. Hopefully, the E-line element approximation routine to be discussed later in the text will minimize the reflections within the line.

Basic transmission line theory has been presented in this chapter along with some pulse forming networks which approximate the open-ended lossless transmission line. It should be remembered that a network composed of a finite number of lumped parameter elements cannot exactly simulate the transmission line which in reality has distributed parameters.

III. A State Variable Model of the E-Line

State equations, a definition of state, and the solution to the state equations are presented in this chapter along with the set of state equations that describes the E-line PFN. In the next chapter the state equations will be used to build an analog model of the E-line and a parameter estimation algorithm which estimates the correct capacitor and inductor values to give a rectangular pulse.

State and State Equations

"The state of a system is a mathematical structure containing a set of n variables $X_1(t)$, $X_2(t)$, $\cdots$, $X_i(t)$, $\cdots$, $X_n(t)$, called state variables, such that the initial values $X_i(t_o)$ of this set and the system inputs $U_j(t)$ are sufficient to uniquely describe the system's future response for $t \geq t_o$" (Ref 1:23-24). The E-line of interest has no inputs, all initial conditions are zero except for the initial voltage on the source capacitor, and physical variables are chosen as the state variables.

The mathematical structure mentioned in the above quotation (Ref 1:23-24) is a set of first order differential equations called state equations. These equations, with the initial condition on the source capacitor, are able to completely describe the PFN's output time response.

Physical Variables. The selection of physical variables as the state variables is based upon the energy storage elements in the PFN (see Table I). The variables must

TABLE I

Energy Storage Elements (Ref 1:25)

Element	Energy	Physical variable
Capacitor C	$\frac{Cv^2}{2}$	Voltage v
Inductor L	$\frac{Li^2}{2}$	Current i

be independent, that is, they cannot be expressed in terms of the remaining physical variables. Since there is a source capacitor at the front of the E-line and two storage elements in each mesh, then there are 2n + 1 physical variables where n is the number of meshes.

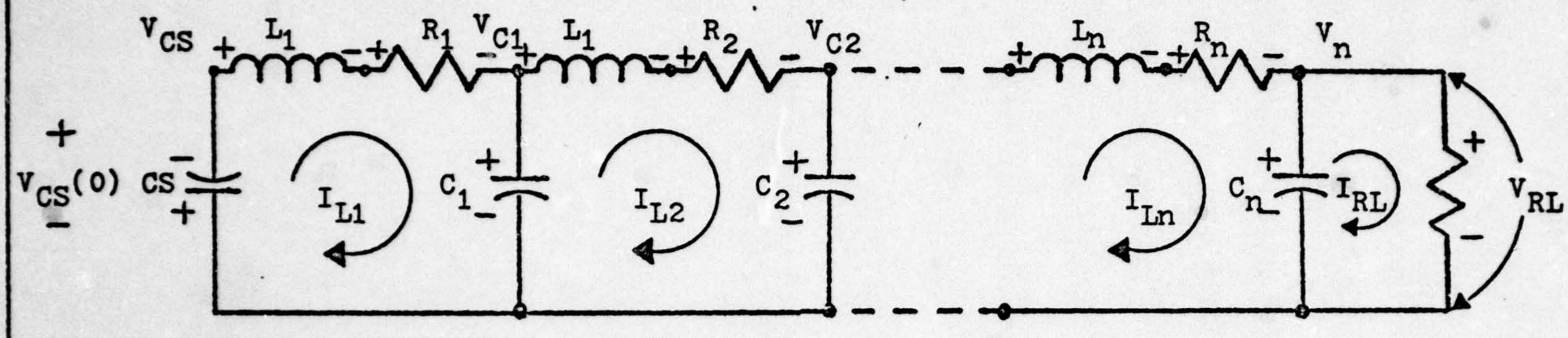
<u>State Equations</u>. Since there are 2n + 1 state variables there must be 2n + 1 state equations. Since this is a first attempt at deriving state equations for the E-line, mutual inductance in the network has been neglected. For simplification, the derivative of a variable is signified by putting a dot over the variable.

The state equations for the E-line are the loop-current and node-voltage equations of each mesh (see Figure 7). For example, the voltage across CS is

$$V_{CS} = \frac{-1}{CS} \int_0^t I_{L1}\,dt + V_{CS}(0) \qquad (2)$$

or the state equation is

$$\boxed{\dot{V}_{CS} = \frac{-1}{CS} I_{L1}} \qquad (3)$$



$i=1,2,3,\cdots n$

where:

L_i is the inductor in the i^{th} mesh

C_i is the capacitor in the i^{th} mesh

R_i is the resistor in the i^{th} mesh

I_{Li} is the current in the i^{th} mesh

V_{Ci} is the voltage across C_i

CS is the charging capacitor

RL is the load resistor

$V_{CS}(0)$ is the initial voltage on CS

Note: notice the sign convention on $V_{CS}(0)$ is the opposite of that on CS

Figure 7. The E-Line Pulse-Forming Network

Notice the initial condition, $V_{CS}(0)$.

The current in inductor L_1 is

$$I_{L1} = \frac{1}{L_1} \int_0^t [V_{CS} - R_1 I_{L1} - V_{C1}]dt \tag{4}$$

or the state equation is

$$\boxed{\dot{I}_{L1} = \frac{V_{CS}}{L_1} - \frac{R_1 I_{L1}}{L_1} - \frac{V_{C1}}{L_1}} \tag{5}$$

The state equation for V_{C1} is

$$\boxed{\dot{V}_{C1} = \frac{1}{C_1}(I_{L1} - I_{L2})} \tag{6}$$

The state equations for the rest of the L's and C's then become

$$\dot{I}_{L2} = \frac{V_{C1}}{L_2} - \frac{R_2 I_{L2}}{L_2} - \frac{V_{C2}}{L_2} \tag{7}$$

$$\dot{V}_{C2} = \frac{1}{C_2}(I_{L2} - I_{L3}) \tag{8}$$

$$\dot{I}_{L3} = \frac{V_{C2}}{L_3} - \frac{R_3 I_{L3}}{L_3} - \frac{V_{C3}}{L_3} \tag{9}$$

$$\dot{V}_{C3} = \frac{1}{C_3}(I_{L3} - I_{L4}) \tag{10}$$

$$\vdots$$

$$\boxed{\dot{I}_{L(n-1)} = \frac{V_{C(n-2)}}{L_{(n-1)}} - \frac{R_{(n-1)} I_{L(n-1)}}{L_{(n-1)}} - \frac{V_{C(n-1)}}{L_{(n-1)}}} \tag{11}$$

$$\boxed{\dot{V}_{C(n-1)} = \frac{1}{C_{(n-1)}}(I_{L(n-1)} - I_{Ln})} \tag{12}$$

$$\boxed{\dot{I}_{Ln} = \frac{V_{C(n-1)}}{L_n} - \frac{R_n I_{Ln}}{L_n} - \frac{V_{Cn}}{L_n}} \tag{13}$$

$$\boxed{\dot{V}_{Cn} = \frac{1}{C_n}\left(I_{Ln} - \frac{V_{Cn}}{RL}\right)} \tag{14}$$

where n is the number of meshes.

The state equations can be expressed in matrix form as

$$\dot{\underline{X}} = A\underline{X} \tag{15}$$

where

$$\dot{\underline{X}} = \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \cdot \\ \cdot \\ \dot{X}_m \end{bmatrix}$$ an mx1 column vector of state variable derivatives

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdot & \cdot & \cdot & a_{1m} \\ a_{21} & a_{22} & \cdot & \cdot & \cdot & \cdot \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ a_{m1} & a_{m2} & \cdot & \cdot & \cdot & a_{mm} \end{bmatrix}$$ an mxm plant coefficient matrix

$$\underline{X} = \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_m \end{bmatrix} \quad \text{an mx1 column vector of state variables}$$

and

$m = 2n + 1$

n = number of meshes

The output equation can then be expressed as

$$Y = \underline{C}^T \underline{X} \qquad (16)$$

where

$$\underline{C}^T = [C_1 \ C_2 \ . \ . \ . \ C_m] \quad \text{an 1xm output row vector}$$

The E-line's state equations expressed in matrix form becomes Equation (18) shown at the top of the next page. Since the voltage on the load is the primary interest, the output equation becomes

$$Y = [000 \cdot \cdot \cdot 01] \begin{bmatrix} V_{CS} \\ I_{L1} \\ V_{C1} \\ \cdot \\ \cdot \\ \cdot \\ I_{Ln} \\ V_{Cn} \end{bmatrix} \qquad (17)$$

which is a scalar.

$$
\begin{bmatrix} \dot{v}_{CS} \\ \dot{i}_{L1} \\ \dot{v}_{C1} \\ \dot{i}_{L2} \\ \dot{v}_{C2} \\ \dot{i}_{L3} \\ \dot{v}_{C3} \\ \cdot \\ \cdot \\ \cdot \\ \dot{I}_{L(n-1)} \\ \dot{v}_{C(n-1)} \\ \dot{I}_{Ln} \\ \dot{v}_{Cn} \end{bmatrix}
=
\begin{bmatrix}
0 & \frac{-1}{CS} & 0 & & & & & & & \\
\frac{1}{L_1} & \frac{-R_1}{L_1} & \frac{-1}{L_1} & 0 & & & & & & \\
0 & \frac{1}{C_1} & 0 & \frac{-1}{C_1} & 0 & & & & & \\
 & 0 & \frac{1}{L_2} & \frac{-R_2}{L_2} & \frac{-1}{L_2} & 0 & & & & \\
 & & 0 & \frac{1}{C_2} & 0 & \frac{-1}{C_2} & 0 & & & \\
 & & & 0 & \frac{1}{L_3} & \frac{-R_3}{L_3} & \frac{-1}{L_3} & 0 & & \\
 & & & & 0 & \frac{1}{C_3} & 0 & \frac{-1}{C_3} & 0 & \\
 & & & & \cdot & \cdot & \cdot & \cdot & \cdot & \\
 & & & & \cdot & \cdot & \cdot & \cdot & \cdot & \\
 & & & & \cdot & \cdot & \cdot & \cdot & \cdot & \\
 & & & & 0 & \frac{1}{L_{(n-1)}} & \frac{-R_{(n-1)}}{L_{(n-1)}} & \frac{-1}{L_{(n-1)}} & 0 & \\
 & & & & & 0 & \frac{1}{C_{(n-1)}} & 0 & \frac{-1}{C_{(n-1)}} & 0 \\
 & & & & & & 0 & \frac{1}{L_n} & \frac{-R_n}{L_n} & \frac{-1}{L_n} \\
 & & & & & & & 0 & \frac{1}{C_n} & \frac{-1}{C_n R_L}
\end{bmatrix}
\begin{bmatrix} v_{CS} \\ I_{L1} \\ v_{C1} \\ I_{L2} \\ v_{C2} \\ I_{L3} \\ v_{C3} \\ \cdot \\ \cdot \\ \cdot \\ I_{L(n-1)} \\ v_{C(n-1)} \\ I_{Ln} \\ v_{Cn} \end{bmatrix}
\tag{18}
$$

$$\underline{\dot{X}} = A\underline{X}$$

The state variables and state equations developed here are not the only ones that can describe the E-line; however, these variables and equations are more readily understandable and logically derived than some of the other forms. If there had been any inputs on the PFN (inputs do not include initial conditions), the state and matrix form of the state equations would have been more complicated; for information on this situation see Ref 1:14-28.

Solving the Output Equation. For a system which has only initial conditions and the initial time, t_o, is taken as zero, the solution to the output equation is

$$Y(t) = \underline{C}^T \exp\,[At]\underline{X}(0) \tag{19}$$

where

$\underline{C}^T$ = the 1xm output row vector

A = the mxm coefficient matrix

$$\exp\,[At] = I + \frac{(At)}{1!} + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \ldots \frac{(At)^k}{k!}$$

= an mxm matrix called the "state transition matrix"

I = the identity matrix

$$\underline{X}(0) = \begin{bmatrix} X_1(0) \\ X_2(0) \\ X_3(0) \\ \cdot \\ \cdot \\ \cdot \\ X_m(0) \end{bmatrix}$$ an mx1 vector containing the initial conditions of the state variables

When this equation is applied to the E-line, the $\underline{C}^T$ vector is given in Eq (18), the A matrix is given in Eq (19), and the initial condition vector, $\underline{X}(0)$, is

$$\underline{X}(0) = \begin{bmatrix} V_{CS}(0) \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 0 \end{bmatrix}$$

Note that the only initial condition in the network is the initial voltage on the source capacitor, and the solution to the output equation is actually the voltage across the load.

The next two chapters will use this solution in an attempt to find the inductor and capacitor values needed for the E-line to produce a rectangular voltage pulse across the load.

There are many methods, other than the one above, for evaluating the state transition matrix. If the A matrix becomes larger than 3x3, evaluating the state transition matrix by hand can be quite a task; therefore, many computer routines have been developed. The subroutine MEXP (Ref 6:1.13), presented in the next chapter, is used in the inductor and capacitor approximation algorithm to solve for the state transition matrix.

MEXP (Ref 6:1.13) is a subroutine developed at the University of Connecticut's School of Engineering. The subroutine solves the state transition matrix using a Chebyshev approximation rather than the series expansion technique presented here.

The information in this chapter only touches the surface of the state variable and state equation concepts. Most of the material in this chapter came from the book, <u>Linear Control System Analysis and Design</u> (Ref 1:14-85), however, this material can be found in almost any introductory book on modern control theory. This material will be used extensively in the next chapter.

IV. Digital Simulation and Parameter Estimation Algorithms for the E-Line Pulse-Forming Network

The first section in this chapter begins by building an analog model of the E-line. Then a simulation algorithm will be developed using a computer subroutine which approximates an analog computer, RKFOUR (see Appendix A). The computer program formed from the simulation algorithm will be used in the next chapter to analyze the PFN as the values of the inductors and capacitors within the network change. In the second section a parameter estimator algorithm is developed which estimates the inductor and capacitor values needed in the E-line for the network to produce a rectangular output voltage pulse. The estimation algorithm uses a function minimization subroutine, ZXMIN, from the International Mathematics and Statistical Libraries, (Ref 5:ZXMIN1-ZXMIN3). The computer programs formed from the simulation and parameter estimation algorithms can be found in Appendix A and Appendix B, respectively.

Analog-Digital Simulation Algorithm of the E-Line

As in the state equations, loop-current and node-voltage equations will be used to develop an analog model of the PFN. However, first order integral equations rather than first order differential equations are needed.

Analog integrator and summing elements are shown in Figure 8. When making an analog model of the E-line, the input to a particular integrator is either the voltage

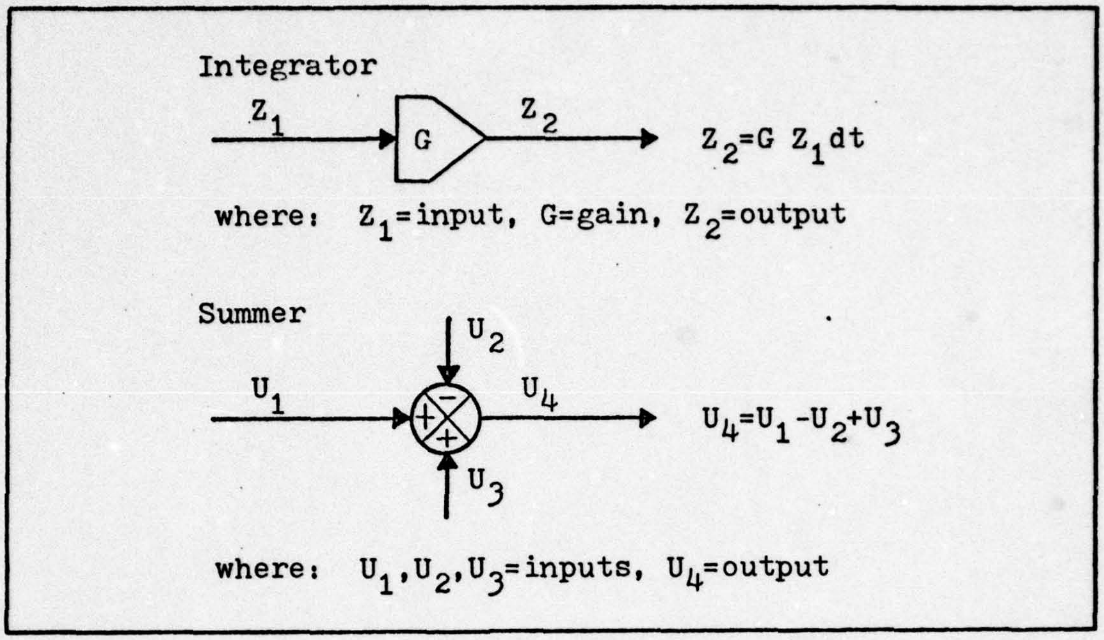


Figure 8. Analog Integrating and Summing Elements

across an inductor or the current through a capacitor. The output of a particular integrator is either the current through an inductor or the voltage across a capacitor.

In Figure 7 the voltage across the source capacitor is given by

$$\boxed{V_{CS} = \frac{-1}{CS} \int_{0}^{t} I_{L1} dt + V_{CS}(0)} \qquad (20)$$

The current in the inductor, L_1, is

$$\boxed{I_{L1} = \frac{1}{L_1} \int_{0}^{t} (V_{CS} - R_1 I_{L1} - V_{C1}) dt} \qquad (21)$$

The equations for the rest of the network become

$$\boxed{V_{C1} = \frac{1}{C_1} \int_0^t (I_{L1} - I_{L2})} \tag{22}$$

$$\boxed{I_{L2} = \frac{1}{L_2} \int_0^t (V_{C1} - R_2 I_{L2} - V_{C2})dt} \tag{23}$$

$$\boxed{V_{C2} = \frac{1}{C_2} \int_0^t (I_{L2} - I_{L3})dt} \tag{24}$$

$$\vdots$$

$$\boxed{I_{L(n-1)} = \frac{1}{L_{(n-1)}} \int_0^t (V_{C(n-2)} - R_{(n-1)} I_{L(n-1)} - V_{C(n-1)})dt} \tag{25}$$

$$\boxed{V_{C(n-1)} = \frac{1}{C_{(n-1)}} \int_0^t (I_{L(n-1)} - I_{Ln})dt} \tag{26}$$

$$\boxed{I_{Ln} = \frac{1}{L_n} \int_0^t (V_{C(n-1)} - R_n I_{Ln} - V_{Cn})dt} \tag{27}$$

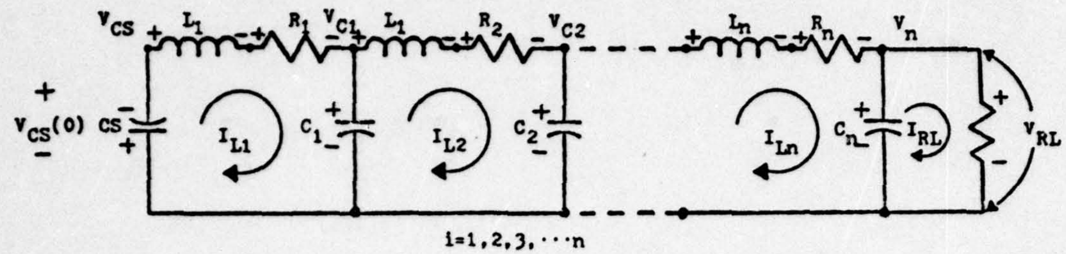
$$\boxed{V_{Cn} = \frac{1}{C_n} \int_0^t (I_{Ln} - \frac{V_{Cn}}{RL})dt} \tag{28}$$

When the equations are put in analog diagram form, the diagram turns out to be that of Figure 9.

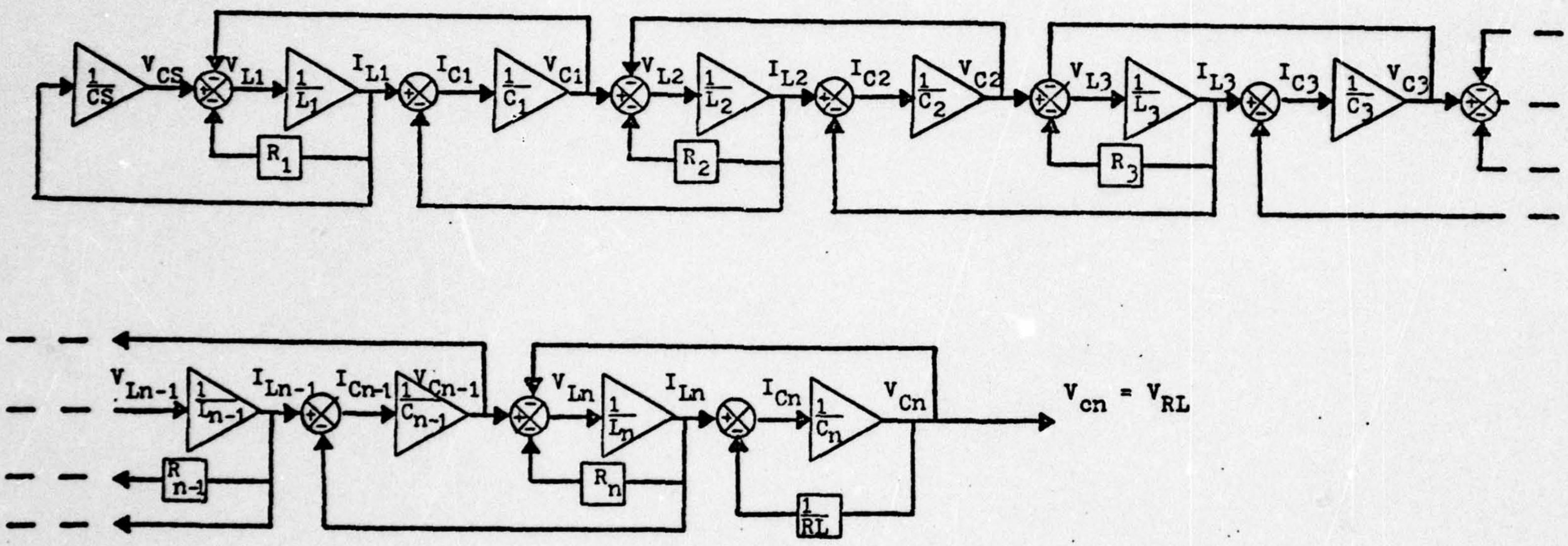
The subroutine RKFOUR (see Appendix A) is a computer routine that simulates an analog computer. The equations used to form the analog diagram of Figure 9 will be put into the program and the performance of the E-line will be analyzed. B.D. Weathers developed RKFOUR while at the University of Missouri, Columbia Campus.

The program that calls RKFOUR has been set up to handle varying pulse rates, varying numbers of meshes, and varying initial voltages on the source capacitor so that E-line specifications (1), (4), and (9) on page 11 can be met. However, getting the energy transferred to the load, having a rectangular pulse of correct duration, and having a 30,000 volt pulse height are dependent on choosing the correct L's and C's in the network.

A switching technique is used to help get the energy from the source capacitor to the load. Some of the energy in the network starts going back into the source capacitor, CS, if while CS is discharging its energy into the network, the current in inductor L_1 goes negative. The recharging of CS is due to reflections in the network. The reflections cause the discharge of CS to be slow and a 20-30 microsecond pulse width is not achievable. To at least prevent the reflections from getting back to CS, the switch between it and L_1 opens when the current in inductor L_1 starts to go negative. When the voltage across CS becomes larger than the



The actual network



Analog model of the actual network

Figure 9. An Analog Model of the E-Line PFN

voltage across capacitor C_1, and the current would normally start going positive again, the switch is closed. This switching shortens the amount of time needed to discharge CS into the load. When the voltage across CS reaches a predetermined value and the current again starts to go negative, the capacitor is removed and recharged for the next pulse while the rest of the network is allowed to run. The switching occurs after each pulse during the entire pulse train. The switching has also been programmed into the main program that calls RKFOUR.

Appendix A contains a flowchart, the operating instructions, and a computer listing of RKFOUR. In the operating instructions and computer printout there should be enough information to understand how RKFOUR works and how it is used.

Parameter Estimation Algorithm

It would be nice if a computer routine, rather than trial and error, could be used to find the inductor and capacitor values that will, as close as possible, approximate the desired rectangular voltage pulse at the output. In this section a computer program is developed that uses a least squares fit objective function in a function minimizing subroutine, ZXMIN (Ref 5:ZXMIN1-ZXMIN3), to determine the L's and C's needed to give the rectangular pulse. Since initial methodology is being developed here, the estimation algorithm only tries to approximate a rectangular voltage pulse for the first pulse in a series of pulses. The first

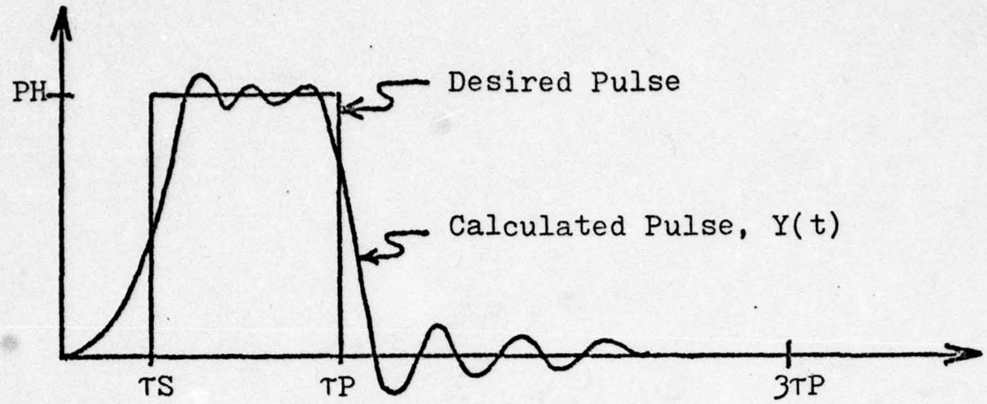
pulse is approximated because the simplest case is when all initial conditions are zero except for $V_{CS}(0)$. Hopefully, if a rectangular pulse is achieved for the first pulse then the rest of the pulses in the series will also be rectangular. A more desirable pulse to approximate would be a pulse later in the pulse train after the prepulse initial conditions have reached steady state.

ZXMIN is a subroutine which minimizes a function of n variables using a quasi-Newton algorithm. The function to be minimized in this application is the least squares fit objective function and the variables are the inductor and capacitor values in the network. ZXMIN calls the subroutine that contains the least squares fit objective function, FUNCT, and by trying different L's and C's minimizes the function. A printout of the operating instructions for ZXMIN is in Appendix B.

The E-line's desired output voltage pulse is shown in Figure 10 with what the actual output voltage pulse might look like. The pulse does not start at time $\tau S = 0$ because there is a finite time before the voltage on the source capacitor can get to the load. It is desirable for the actual pulse to not only approximate the desired pulse but go to zero after time τP.

The least squares fit objective function is defined as

$$\boxed{F = \int_{\tau S}^{\tau P} (PH - Y(t))^2 dt + K_1 \int_{\tau P}^{3\tau P} Y(t)^2 dt} \qquad (29)$$



where:

$Y(t)$ = the calculated output voltage pulse

PH = pulse height

τS = time when desired pulse starts

τP = time when desired pulse goes to zero

3τP = time when oscillations in actual pulse should have stopped

Figure 10. Desired and Actual Output Pulses

where

K_1 = weighting factor

When F is a minimum the calculated pulse approximates the desired pulse as closely as possible while keeping the oscillations after the pulse to a minimum. The oscillations after the pulse are due to continued reflections within the E-line. The weighting factor, K, determines how desirable it is to keep the postpulse oscillations small. If K_1 is small, $0 \leq K_1 \leq 0.5$, then approximating the pulse is more desirable than the postpulse oscillations. If K_1 is large, $0.5 < K_1 \leq 1.0$, then the postpulse oscillations are at least as desirable as approximating the pulse.

$Y(t)$ in Eq (29) is the output voltage, V_{RL} of Figure 7.

The object of minimizing Eq (29) is to make the voltage approximate the desired rectangular pulse. Recall from Eq (19) of Chapter III that Y(t) is also known as the solution to the output equation, i.e.,

$$Y(t) = V_{RL} = \underline{C}^T \exp[At]\underline{X}(0) \tag{30}$$

where, for the E-line:

$\underline{C}^T$ = the 1xm output row vector [00···01]
exp [At] = the mxm state transition matrix
$\underline{X}(0)$ = the mx1 initial condition vector

$$= \begin{bmatrix} V_{CS}(0) \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

Therefore, the solution to the output equation can be, and is, used for Y(t) in the least squares fit objective function.

The integrals in Eq (29) are done numerically using the trapezoidal rule (Ref 4:144-148). According to the trapezoidal rule, to evaluate the integral

$$I = \int_a^b f(t)dt \tag{31}$$

one can divide the interval, $a \leq t \leq b$, into n equal subintervals each of width Δt (see Figure 11), where

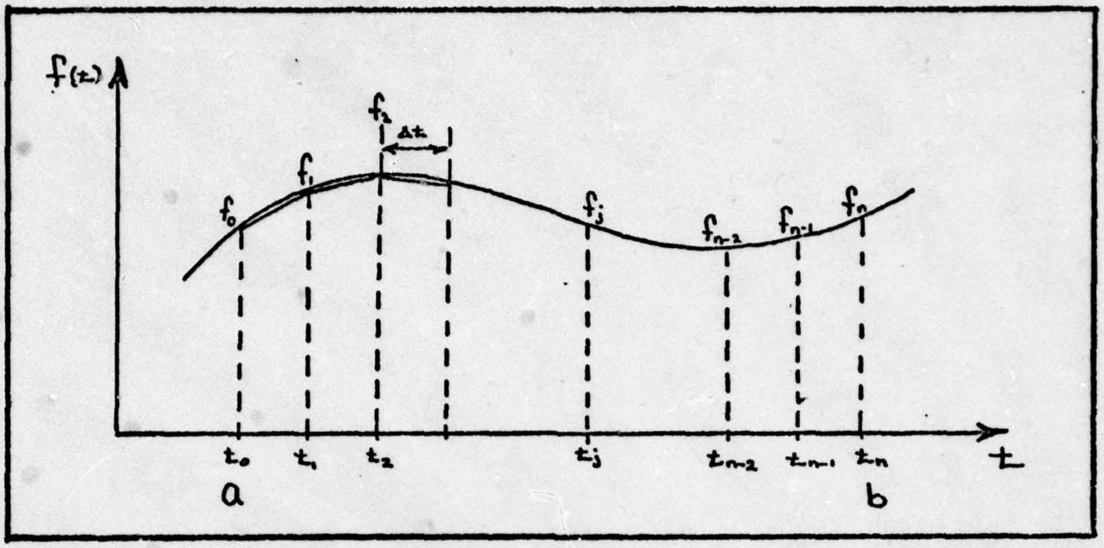


Figure 11. Approximating an Integral

$$\Delta t = \frac{b-a}{n} \tag{32}$$

and the integral is then approximated by

$$I = \int_a^b f(t)dt \approx \frac{\Delta t}{2}(f_0+2f_1+2f_2+\cdots+2f_{n-2}+2f_{n-1}+f_n) \tag{33}$$

$$\approx \frac{\Delta t}{2}\ f_0+f_n+2\sum_{j=1}^{n-1} fj$$

where

$f_0, f_1, f_2, \cdots f_{n-1}, f_n$ are the values of the function at the points $t_0, t_1, t_2 \cdots t_{n-1}, t_n$

As Δt gets smaller, the approximation to the true integral becomes more accurate.

When applying the trapezoidal rule to the least squares

fit objective function, Eq (29), the result is

$$F = \frac{\Delta t}{2}\left\{(PH - Y(\tau S))^2 + \left(2\sum_{i=\frac{\tau S}{\Delta t}+1}^{\frac{\tau P}{\Delta t}} (PH - Y(i\cdot\Delta t))^2\right)\right.$$

$$\left. + \left(2K_1 \sum_{i=\frac{\tau P}{\Delta t}+1}^{\frac{3\tau P}{\Delta t}-1} (Y(i\cdot\Delta t))^2\right) + (Y(3\tau P))^2\right\} \qquad (34)$$

Notice that $\frac{\tau S}{\Delta t}$, $\frac{\tau P}{\Delta t}$, and $\frac{3\tau P}{\Delta t}$ must come out to be integers. When substituting Eq (30) for $Y(\tau S)$, $Y(i\cdot\Delta t)$, and $Y(3\tau P)$ Eq (34) becomes

$$F = \frac{\Delta t}{2}\left\{(PH - \underline{C}^T \exp[A\cdot\tau S]\underline{X}(0))^2\right.$$

$$+ \left(2\sum_{i=\frac{\tau S}{\Delta t}}^{\frac{\tau P}{\Delta t}} (PH - \underline{C}^T \exp[A\cdot i\cdot\Delta t]\underline{X}(0))^2\right)$$

$$+ \left(2K_1 \sum_{i=\frac{\tau P}{\Delta t}-1}^{\frac{3\tau P}{\Delta t}-1} (\underline{C}^T \exp[A\cdot i\cdot\Delta t]\underline{X}(0))^2\right)$$

$$\left. + (\underline{C}^T \exp[A\cdot 3\tau P]\underline{X}(0))^2\right\} \qquad (35)$$

If the state transition matrix is found for time Δt, i.e.,

$$\exp[A\Delta t] = I + \frac{A\Delta t}{1!} + \frac{A^2(\Delta t)^2}{2!} + \frac{A^3(\Delta t)^3}{3!} + \ldots \frac{A^k(\Delta t)^k}{k!} \qquad (36)$$

$= \text{an } m \times m \text{ matrix}$

the values for $\exp[A\tau S]$, $\exp[Ai \cdot \Delta t]$, and $\exp[A3\tau P]$ can be found easily by bringing the value of $\exp[A\Delta t]$ to the correct power, i.e.,

$$\exp[A \cdot \tau S] = \exp\left[(A\Delta t) \cdot \frac{\tau S}{\Delta t}\right] \tag{37}$$

$$\exp[Ai \cdot \Delta t] = \exp[(A\Delta t) \cdot i] \tag{38}$$

and

$$\exp[A \cdot 3\tau P] = \exp\left[(A\Delta t) \cdot \frac{3\tau P}{\Delta t}\right] \tag{39}$$

The subroutine that contains the least squares fit objective function, FUNCT, calls the subroutine MEXP (see page 22) which calculates the state transition matrix $\exp[A \cdot \Delta t]$.

Earlier in this section it was stated that the subroutine ZXMIN tries to minimize the subroutine that contains the least squares fit objective function, actually ZXMIN tries to minimize Eq (35), an approximation to the least squares fit objective function. Appendix B contains a flow chart of FUNCT, operating instructions for ZXMIN and MEXP, a computer printout of the main program which calls ZXMIN, and a computer printout of subroutine FUNCT.

In this chapter the algorithms needed to form the analog approximation routine and the parameter estimation routine were developed. The next chapter uses the programs to compare sets of L's and C's and predict trends for the L's and C's to get the desired rectangular output voltage pulse.

V. Analysis of the Simulation's Results

The analog approximation routine described in the previous chapter has been set up to simulate the voltage and current time responses for the E-line. The values for the inductors and capacitors predicted by the parameter estimator are put into the simulation program with several other sets of inductor and capacitor values. The output voltage calculated from the simulation program is plotted on graphs for the different sets of inductor and capacitor values. Hopefully, the graphs will aid in predicting a trend for the inductors and capacitors to follow in achieving the desired rectangular pulse.

The E-line for which all data is based has

1. 5 meshes.
2. Source capacitance, CS, of 1.33×10^{-4} farads.
3. A pulse repetition rate of 100 pulses per second.
4. A source capacitor initial voltage of 30,000 volts.

The desired pulse is delayed by 5 microseconds, has a pulse width of 20 microseconds, and has a pulse height of 30,000 volts, i.e.,

$$TS = 5 \times 10^{-6} \text{ sec}$$

$$TP = 25 \times 10^{-6} \text{ sec}$$

$$PH = 30{,}000 \text{ volts}$$

Data on several more E-line and desired pulse cases would have been collected but time was not permitting.

L and C Values Estimated by ZXMIN

When the preceding values for τS, τP, and PH are used in the least squares fit objective function and the weighting factor, K_1, is chosen as 0.5, the parameter estimation routine predicts the capacitances and inductances given in row 1 of Table II. When these parameter values are put into the E-line, the resulting output voltage, for the weighting factor chosen, should approximate as close as possible the desired rectangular pulse.

Base Values for L and C

In the book Pulse Generators (Ref 7:201) a set of inductor and capacitor values has been calculated which gives an approximately rectangular output voltage pulse if a conventional line-type pulser charging scheme is used to put energy into the network (see page 3 of this thesis). Although the E-line of interest uses a different type charging scheme, the values calculated in Ref 7 are used as base values for the inductors and capacitors in the network (see Figure 12). Although these values are not the exact values needed by the E-line to produce the desired rectangular pulse, they are a starting point. The base values along with multiples of the base values are found in Table II. Because the last capacitor in the network, C_5, is important in determining the time constant at the load, C_5 should not be changed; thus, C_5 stays the base value in all multiples of the base values.

TABLE II

Test Cases put in the Simulation Program

TEST CASES	INDUCTOR AND CAPACITOR VALUES FOR EACH CASE									
	C_1	C_2	C_3	C_4	C_5	L_1	L_2	L_3	L_4	L_5
Parameter Estimator L&C	7.829E-06	7.779E-06	8.515E-06	1.079E-05	2.029E-05	2.150E-07	5.244E-08	1.516E-07	1.639E-09	1.270E-07
Base C and Base L	7.830E-06	7.782E-06	3.521E-06	1.079E-05	2.029E-05	2.577E-07	2.086E-07	2.171E-07	2.554E-07	3.607E-07
Base C and .5 Base L	7.830E-06	7.782E-06	8.521E-06	1.079E-05	2.029E-05	1.289E-07	1.093E-07	1.086E-07	1.277E-07	1.804E-07
Base C and 2 Base L	7.830E-06	7.782E-06	3.521E-06	1.079E-05	2.029E-05	5.159E-07	4.172E-07	4.342E-07	5.108E-07	7.214E-07
Base C and 3 Base L	7.830E-06	7.782E-06	8.521E-06	1.079E-05	2.029E-05	7.731E-07	6.258E-07	6.513E-07	7.662E-07	1.036E-06
Base L and .5 Base C	3.915E-06	3.891E-06	4.261E-06	5.395E-06	2.029E-05	2.577E-07	2.086E-07	2.171E-07	2.554E-07	3.607E-07
Base L and 2 Base C	1.566E-05	1.556E-05	1.704E-05	2.158E-05	2.029E-05	2.577E-07	2.086E-07	2.171E-07	2.554E-07	3.607E-07
Base L and 3 Base C	2.349E-05	2.335E-05	2.556E-05	3.236E-05	2.029E-05	2.577E-07	2.086E-07	2.171E-07	2.554E-07	3.607E-07

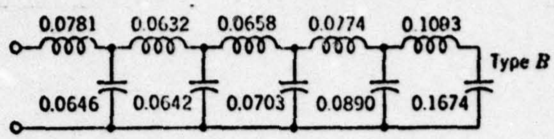


Figure 12. Multiply the values of the inductances by the product RL (Pulse Width) and the values of the capacitances by Pulse Width/RL. The inductances are in henrys and the capacitances in farads. (Ref 7:201)

Analysis of Results

The simulation program has calculated the output voltage for each of the test cases in Table II. Graphs of the output voltage as a function of time can be found in Figures 13, 14, and 15. The desired pulse shape has been drawn on all the graphs, and the graphs have been normalized to the 30,000 volt desired pulse height.

In Figure 13 it can be seen that although the inductor and capacitor values predicted by the parameter estimator do not produce a curve that approximates the desired pulse very well, there is a noticeable improvement over the curve resulting from the base values of L and C. When using the estimator's values the rise time is very good and the first two peaks fall within the desired pulse width; however, the fall time is far too long and the pulse height is not nearly high enough. The base values' voltage curve has a slightly higher first peak, but the second peak falls outside the desired pulse width and is too short; also, the fall time is too long.

In Figure 14 the curves shift to the right and the

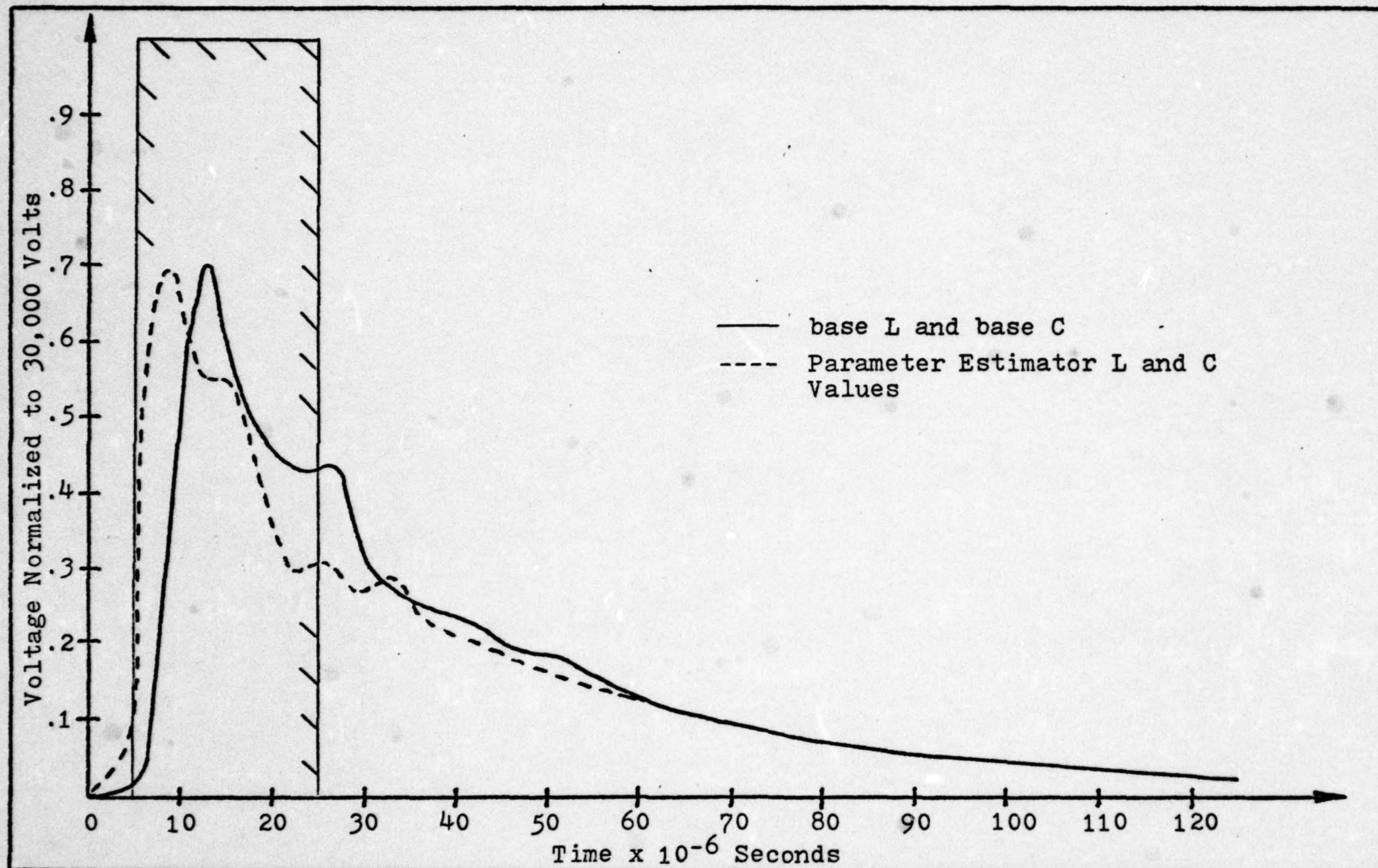


Figure 13. The E-line's output voltage as calculated by the simulation program for different sets of inductor and capacitor values.

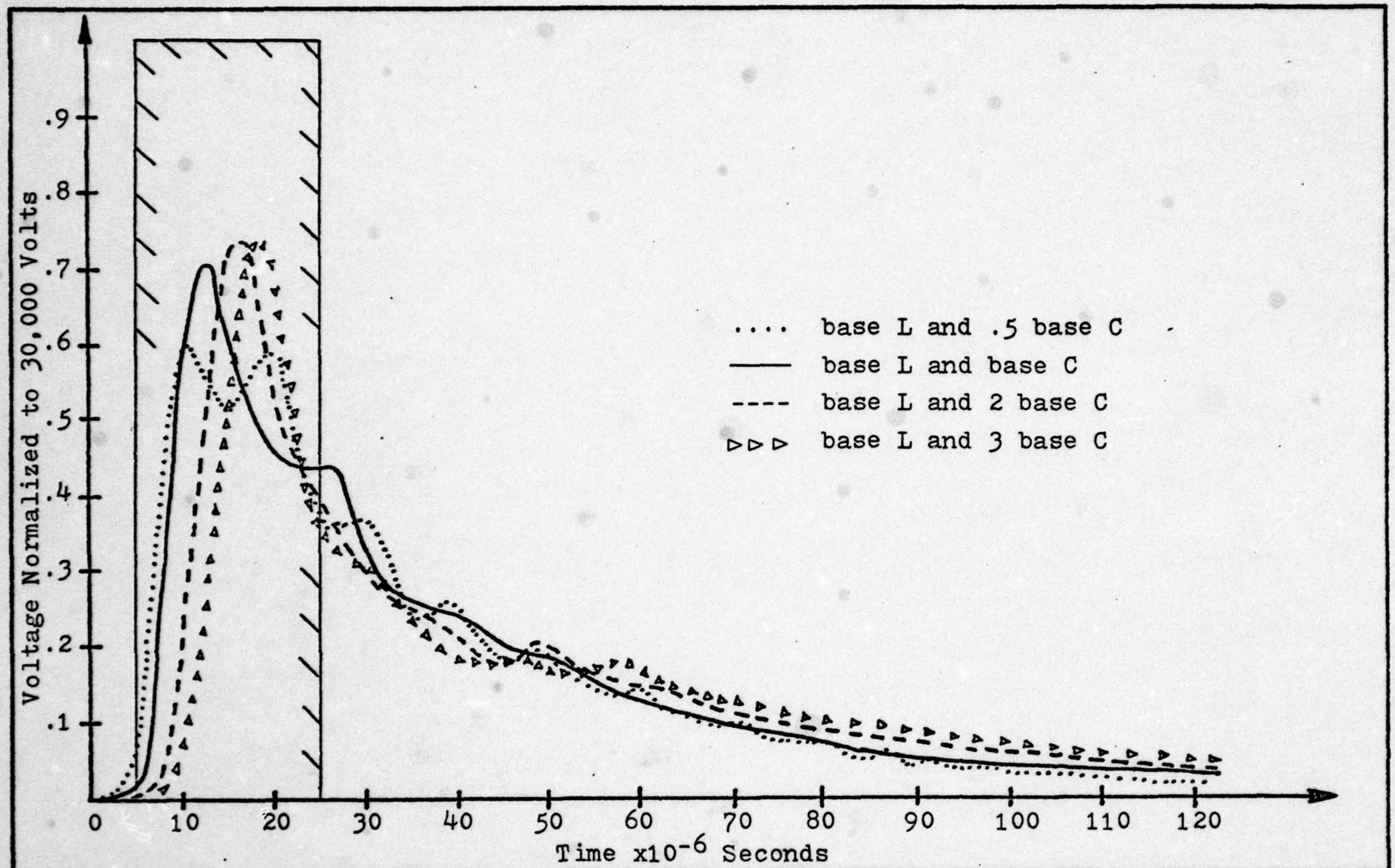


Figure 14. The E-line's output voltage as calculated by the simulation program for different sets of inductor and capacitor values.

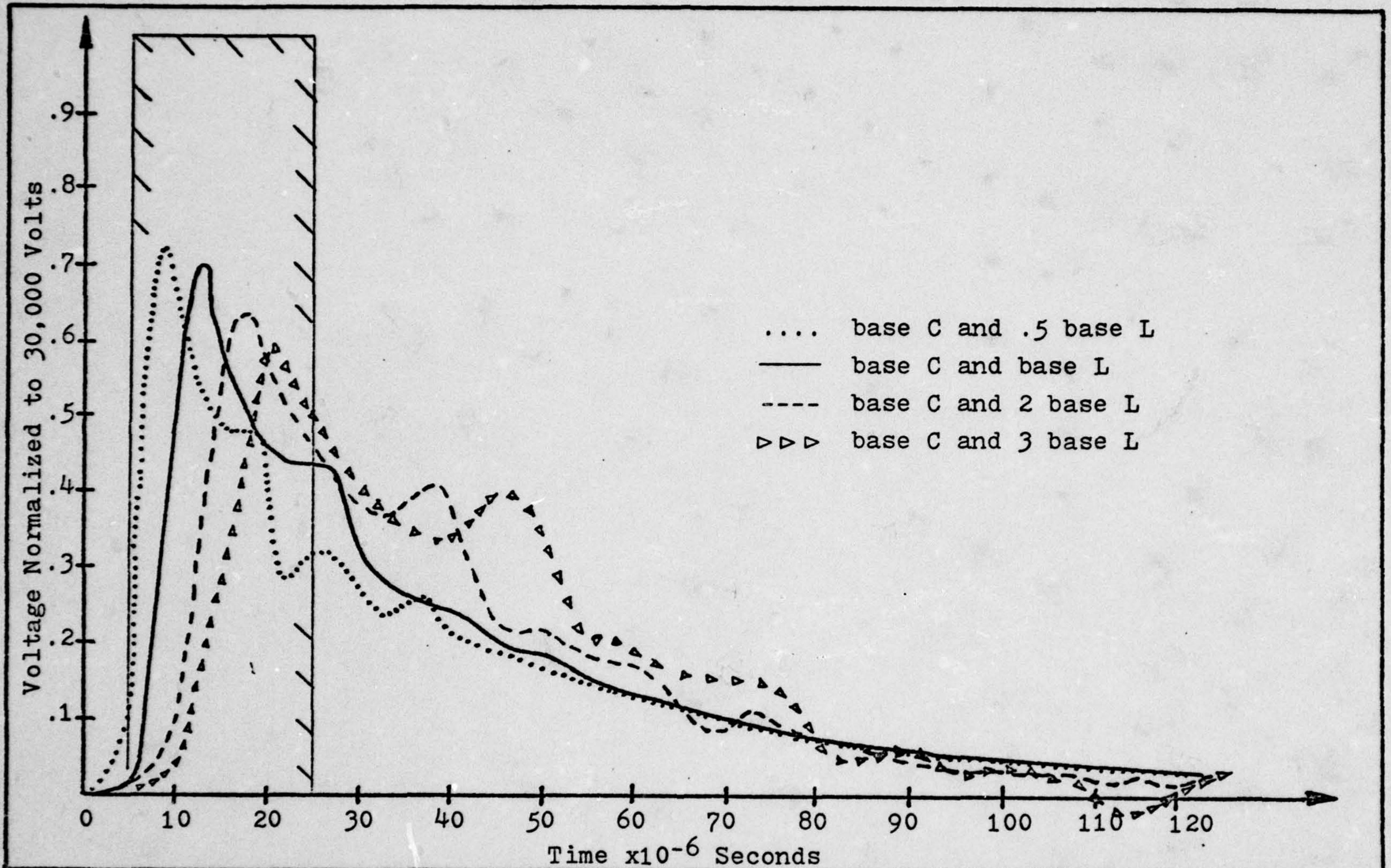


Figure 15. The E-line's output voltage as calculated by the simulation program for different sets of inductor and capacitor values.

height of the second peaks decrease as the capacitor values in the network increase. When the base values for the capacitor are cut in half the corresponding curve has its first two peaks at about the same height and within the desired pulse width, but there are three undesirable oscillations outside the pulse and the fall time is again too long. As the capacitor values are increased above their base value, their corresponding curves are definitely undesirable in that there is only one major peak and the fall time is even longer.

In Figure 15, as the inductor values increase, the curves shift to the right and the height of the first peak decreases. When the base values for the inductors are cut in half, the corresponding curve has a very good rise time, has high first and second peaks, and the fall time between the second and third peaks looks good. However, the curve's overall fall time is too long and there are two undesirable oscillations outside the desired pulse width. As the inductor values are increased it takes too long for the pulse to rise, there are pronounced oscillations after the pulse, and the pulse width has to be increased for most of the energy in the network to get to the load.

None of the curves on Figures 13, 14, or 15 look much like the desired output pulse. When the weighting factor is chosen as 0.5 it is apparent that the parameter estimator did a poor job of predicting the inductances and capacitances needed for the network to approximate the desired pulse.

Changing the weighting factor or making the weighting factor time varying may improve the parameter estimator's prediction.

Trends

Although the curves are somewhat different from the desired pulse, the curves do display some trends. If the inductor values are decreased from their base values, the rise time and fall time can be improved. If the capacitor values are decreased from their base value, except for the last capacitor, the first and second peaks can be made to have the same height. Also, if the inductor values are decreased the first and second peaks can both be higher. However, if the inductor and capacitor values are decreased too much the output voltage will look as though the source capacitor is discharging directly into the load.

If the trends mentioned above are incorporated into a trial and error technique it may be possible to find the inductor and capacitor values needed. Trial and error techniques, however, can be quite time consuming and the time might be better spent on attempting to improve the existing estimation program or develop a new program.

VI. Conclusions and Recommendations

Conclusions

The network simulation program models the discharge, switching action, and pulsing action of the actual network very well and is indispensable in analyzing the output voltage for different sets of inductor and capacitor values. The program is flexible enough to model a variety of changes that may be desired in the actual network.

The parameter estimation program, as applied in this thesis, is not working as well as desired. It may not be working well because of a simple FORTRAN error, but it is more likely due to an error in the estimation algorithm, not describing the weighting factor properly, and/or problems with the subroutines used by the program. Unless the estimation program can be improved or another estimation program can be developed, finding the inductor and capacitor values needed for the E-line to approximate the desired output voltage pulse may involve considerable trial and error. Recommendations are made in the next section on what might be done to improve the parameter estimator or develop a new parameter estimator.

The network's desired output voltage pulse was not achieved with any of the test cases listed in Table II, but the curves corresponding to the test cases, Figures 13, 14, and 15, displayed some trends. Using these trends as a starting point, the desired rectangular pulse may be

achievable through trial and error. However, it may be the case that it is not possible to approximate the desired rectangular pulse very well with the particular E-line pulse-forming network analyzed. In the following section, changes that might be made in the network to get the calculated output voltage pulse are recommended.

Recommendations

Although the simulation program seems to describe the actual output voltage well, including mutual inductance in the program will improve its accuracy. Mutual inductance has a significant impact on the output voltage wave shape (Ref 2). Therefore, if the mutual inductance is included in the simulation program, it may be found that the desired rectangular pulse has been achieved. If this thesis is chosen as a starting point for another thesis, mutual inductance should definitely be included in the simulation program.

As is, the parameter estimator program is doing a poor job of predicting the inductor and capacitor values needed. To improve the estimator's predicting ability, one or more of the following might be tried:

1. Include more of the system's dynamics, such as mutual inductance between meshes.

2. Have the estimator try to approximate a whole series of pulses in the pulse train.

3. Change the value of the weighting factor or make the weighting factor time varying.

4. Make it possible for the estimation program to not only change the inductor and capacitor values, but

also the resistor values when trying to approximate the desired output pulse.

A new parameter estimation algorithm may need to be developed if the present estimator continues to fail after the above recommendations have been investigated. Captain J. Gary Reid, USAF, has a thesis student presently working on a function minimizing algorithm which could be of use in a new estimation program. For more information he can be contacted at AFIT/ENG, Wright-Patterson AFB, Ohio 45433 or AV 785-3450.

As noted, none of the test cases approximated the desired rectangular pulse very well. The network may be changed in one or more of the following ways in an attempt to get a more rectangular output voltage pulse:

1. Increase the number of meshes in the network.

2. To stop postpulse oscillations in the load, put a switch between the network and the load such that when the current in the load goes negative the switch opens.

3. Distribute the stored network energy throughout the network by charging all the capacitors rather than just the source capacitors; this is a conventional line-type pulser and the advantage of a lightweight more reliable system is lost.

4. Put a transformer between the network and the load and have the turns ratio such that the characteristic impedance of the network matches the load impedance; this could decrease the pulse's fall time or reduce the postpulse oscillations.

5. Replace the E-line with a different pulse-forming network.

Bibliography

1. D'Azzo, John J. and Constantine H. Houpis. Linear Control System Analysis and Design. New York: McGraw-Hill Book Company, 1975.

2. Dailey, C. L. and C. W. White. PFN Characterization and Life Test. Technical Report AFAPL-TR-75-69. Wright-Patterson AFB, Ohio: Air Force Aero-Propulsion Laboratory, Air Wright Aeronautical Laboratories, Air Force Systems Command, January 1976.

3. Francis, J. F. High Voltage Pulse Techniques. AFOSR-74-2639-5. Lubbock, Texas: Plasma Laboratory, Department of Electrical Engineering, Texas Tech University, December 1976.

4. Hornbeck, Robert W. Numerical Methods. New York: Quantum Publishers, Inc., 1975.

5. IMSL Library 3, Volume 2. Houston, Texas: International Mathematical and Statistical Libraries, Inc., July 1977.

6. Kleinman, David L. A Description of Computer Programs for use in Linear Systems Studies. Technical Report TR-77-2. Storrs, Connecticut: Department of Electrical Engineering and Computer Science, University of Connecticut, July 1977.

7. Pulse Generators, edited by G. N. Glasoe and J. V. Lebacqz. Massachusetts Institute of Technology Laboratory Series, Volume 5, Editor-in-Chief Louis N. Ridenour. New York: McGraw-Hill Book Company, Inc., 1948.

8. Van Valkenburg, M. E. Network Analysis (Third Edition). Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1974.

Appendix A Digital Simulation Program

Operating Instructions and Flow Chart for RKFOUR

and Simulation Program Source Listing

Operating Instructions for RKFOUR

Programmer: B. D. Weathers

Purpose: RKFOUR is an integration routine designed to provide the equivalent of an analog computer integrator for the solution of normal-form differential equations.

Features

---- An arbitrary number of integrators can be used. The number used is determined by the size of the storage arrays set in the calling program.

---- All variables that affect the operation of RKFOUR appear in the calling statement; multiple uses of the program are possible, but care must be used to prevent time desnchronization.

---- The independent variable step size is determined automatically, but if desired this program can be made to run with a fixed step size.

---- The adaptive step size feature, if used, will insure little trouble with numerical instability.

---- The limits on the adaptive step size may be set in the calling program, or default values may be used.

---- The error limits that determine when the step size should be changed are program parameters, and hence may be externally controlled.

---- An adjustable delay is available to prevent eratic step size changing. Once the criteria is satisfied step size increases by the appropriate factor at each time.

---- Step sizes may be changed externally, but may not be changed in the middle of an iteration, that is when KEEP is equal to 0.

A solution point counter is contained in the routine to enable termination if appropriate independent variable criteria is not available.

Suggestions for Use

Before using this subroutine for the first time, a careful examination of the flow chart should be made. Also a thorough examination of theprogram parameter should be carried out to insure that the default values of the parameters will be satisfactory, if used.

You must set all non zero initial integrator outputs. All non-unity integrator gains must also be set in section 2.

The value for H(1) - Step Size - should be set as close to the predicted sampling interval as possible. This will insure a more efficient operation for the first pass.

If running in "Open Shop" and the 20 second time limit is just a "bit to short," add "RUN=NO CHECK" to the control card. Do this only after your name, after you are SURE the program is free of programming errors. On a large job this can add 15% to the available computation time, by eliminating several compiler tests.

CALL RKFOUR (NINT, Y, X, GA, PE, XX, H, ZZ1, ZZ2, ZZ3, TIME, PNTS, KEEP, IFL)

NINT - Number of integrators. Storage for the integrators must be defined in the calling program. There is no maximum, but the example program is set for up to 20.

Y - An array that contains the integrator output. Default initial values are zero.

X - An array that contains the integrator inputs.

GA - An array that contains constant gains associated with each integrator. Default values are 1.0.

PE - A working array used to store absolute peak values of the integrator outputs. This array is set to zero during initial entry to RKFOUR.

XX - An array used in two ways. If I denotes the Ith integrator, then XX(1,I) stores the past value of the integrator output and XX(2,I) stores the past value of the integrator input. During the integration cycle the XX(3,I) and XX(4,I) positions are used to store the derivative values needed by the integration algarithm. At the end of a cycle, XX(3,I) holds the new integrator output and XX(4,I) holds the new integrator input. It is possible, with the values, to implement a third order interpretation for the values of the integrator outputs between steps. This would serve to offset the problem of obtaining values at discrete time points and would be more efficient than step size manipulation.

H Array - The integration control parameters.

H (1) - The current independent variable step size.

H (2) - Minimum permissible step size (Default value is 0.0).

H (3) - Maximum step size (Default value is 1.0E + 8).

H (4) - Factor used to modify the step size-Must be greater than 1.0 (Default value is SORT of 2=1.414214).

H (5) - Proportional to the lower error bound (Default value is 1.0E - 4).

H (6) - Proportional upper error bound (Default value is 1.0E - 02)

H (7) - Adaptive step size control (Set to -1 for constant step size operation) (Default value of 0.0 cause adaptive step size operation).

H (8) - Delay in increasing the step size after error criteria is met. (Default value is 3.0).

H (9) - Last value of the independent variable.

ZZ1, ZZ2, ZZ3 - Working arrays used to store the constants needed for the integration routine. These arrays are

TIME - The independent variable set within RKFOUR. The default initial value is zero.

PNTS - The counter of permanent solution extensions. Use this variable for termination when other termiation options fail.

KEEP - This variable is set to ONE only when the data in the derivative evaluation section is permanent. It is zero at all other times. This variable should be used as a control upon memory dependent functions.

IFL - The IFLag array (See flow chart)

IFL(1) - Set to -1 at the first entry to RKFOUR, and is 0 at the first exit. On subseqeunt exits it has the value +1.

IFL(2) - Has the value -1 to +3 depending upon where the integration routine is in its operation cycle.

IFL(3) - This flat is used to control the step size from the derivative evaluation section of the calling program. It is set to -1 for step size reduction, and +1 for an increase in the step size.

IFL(4) - This flag is -1 when initialializing calculations are to be made. It is 0 when only the derivative evaluation is required, and is +1 when the solution has been successfully extended over one step of the independent variable.

IFL(5) - This flag is set to +1 immediately after IFL(4) is -1 for the first time. It is intended as a signal for other routines that may be used that initialization is underway.

IFL(6) - This flag is used for the temporary storage of the variable KEEP.

The Basic RKFOUR Program

```
      DIMENSION Y(20)4, X(20)4, GA(20)4, PE(20),XX(4,20),H(9)
      DIMENSION ZZ1(20),ZZ2(20),ZZ3(20),IFL(7)
```

If Other Dimensioned Variables are required the necessary DIMENSION statements go here.

SECTION 1 INSERTIONS GO HERE

```
      NRUN= (Number of runs to be made)
      DO 6 IRUN=1,NRUN
      ISTOP=0
      KEEP=1
      IFL(1)=-1
      NINT= (Number of integrators required)
 1    IFL(2)=-1
 2    CALL RKFOUR(NINT,Y,X,GA,PE,XX,H,ZZ1,ZZ2,ZZ3,TIME,PNTS,KEEP,IFL)
      IF (IFL[4])3,4,5
 3    INT=1
      TIME1=-1.256789
      NC=1
      NCC=1  (Portion of Points released to Section 5)
      FINTIM=  (Maximum Value for TIME in this run)
      PTMAX=  (Maximum Number of solution points for this run)
```

SECTION 2 INSERTIONS GO HERE

```
 4    IF(TIME - TIME1)7,8,9
 7    TIME1=TIME
```

SECTION 3 INSERTIONS GO HERE

```
 8    CONTINUE
```

SECTION 4 INSERTIONS GO HERE (Exclude everyting nonessential)

```
        GO TO 2
 5    IPR=0
      NC=NC-1
      IF (NC)9,9,10
 9    NC=NCC
      IPR=1
10    IF(TIME.GE.FINTIM.OR.PNTS.GE.PTMAX) ISTOP=1
      IF ([IPR+ISTOP].EQ.0) GO TO 11
```

SECTION 5 INSERTIONS GO HERE

```
11    INIT=0
      IF (ISTOP)1,1,6
 6    CONTINUE
      WRITE (6,100)
100   FORMAT (1H1, 1X)
      STOP
      END
```

Basic RKFOUR System

There are a number of variables in the RKFOUR System whose value depends upon the problem undertaken.

NRUN - The number of runs to be made.

NINT - The number of integrators to be used in the run (limited to 20 by storage allocation in the stock system).

NCC - The portion of solution points that are released to the output section (Section 5) NCC = 1 delivers every point, NCC = 2, every second point, etc.

FINTIM - The maximum value that the independent variable will attain. The program will terminate upon the first solution point to meet or exceed this value.

PTMAX - The maximum number of solution points in any one run.

IPNT - The point counter used in plot routines. It may be set to 0 in one of two locations. If summary curves are desired, set to 0, before the IRUN DO statement. If no summary curves are desired, set to 0 after the IRUN DO statement.

INIT - This variable is 1 the first interation step, and 0 thereafter. This can be used as a memory flag for memory dependent functions.

The Basic RKFOUR System contains five sections where the particular problem statements may be inserted.

1. This section is used to initialize those values which are not run dependent, or whose calculation in block form is more efficient. In this case they may be indexed by IRUN for use in the rest of the program.
2. This section is used to:

    a. Set non-zero integrators initial conditions

b. Set non-unity integrator gains

c. Override any default values set by RKFOUR subroutine

d. Carry out calculations that are run dependent and need be carried out only once per run

3. This section is used to carry out all calculations of the functions of the independent variable.

4. This section contains the derivative evaluation equations. It is executed four times for each point and should be carefully structured. For example:

X(1)=(A+B)/(D+E) + U(1)*(G+H) would be efficiently written as X(1) = AB + Y(1) * GH

where AB = (A+B)/(D+E)

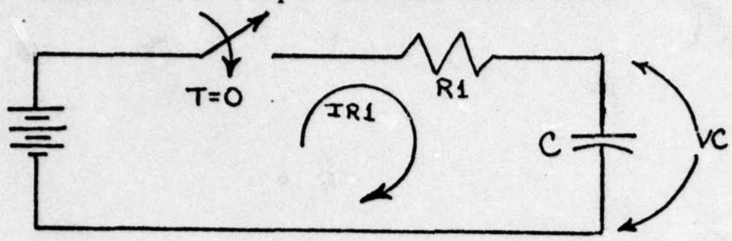
GH = G + H

and are evaluated in section 1,2, or 3 as is appropriate.

5. This section is used to call the output routines. If NCC is not equal to 1 it is not entered on every pass, and this should be considered when structuring the problem.

RKFOUR Example

Consider the simple circuit shown below



From elementary circuit theory we know:

C*VC = 1R1 and VR1 = V = VC

Now let us define the capacitor voltage as a state variable, which leads directly to:

Y(1) - VC

Therefore, with GA(1) = 1.0/C

```
VC=Y(1)
1R1=(V-VC)/R
X(1)=1R1
```

are the derivative evaluation equations.

In the example program on the following pages, the capacitor was originally charged to -1 volt before the switch was closed.

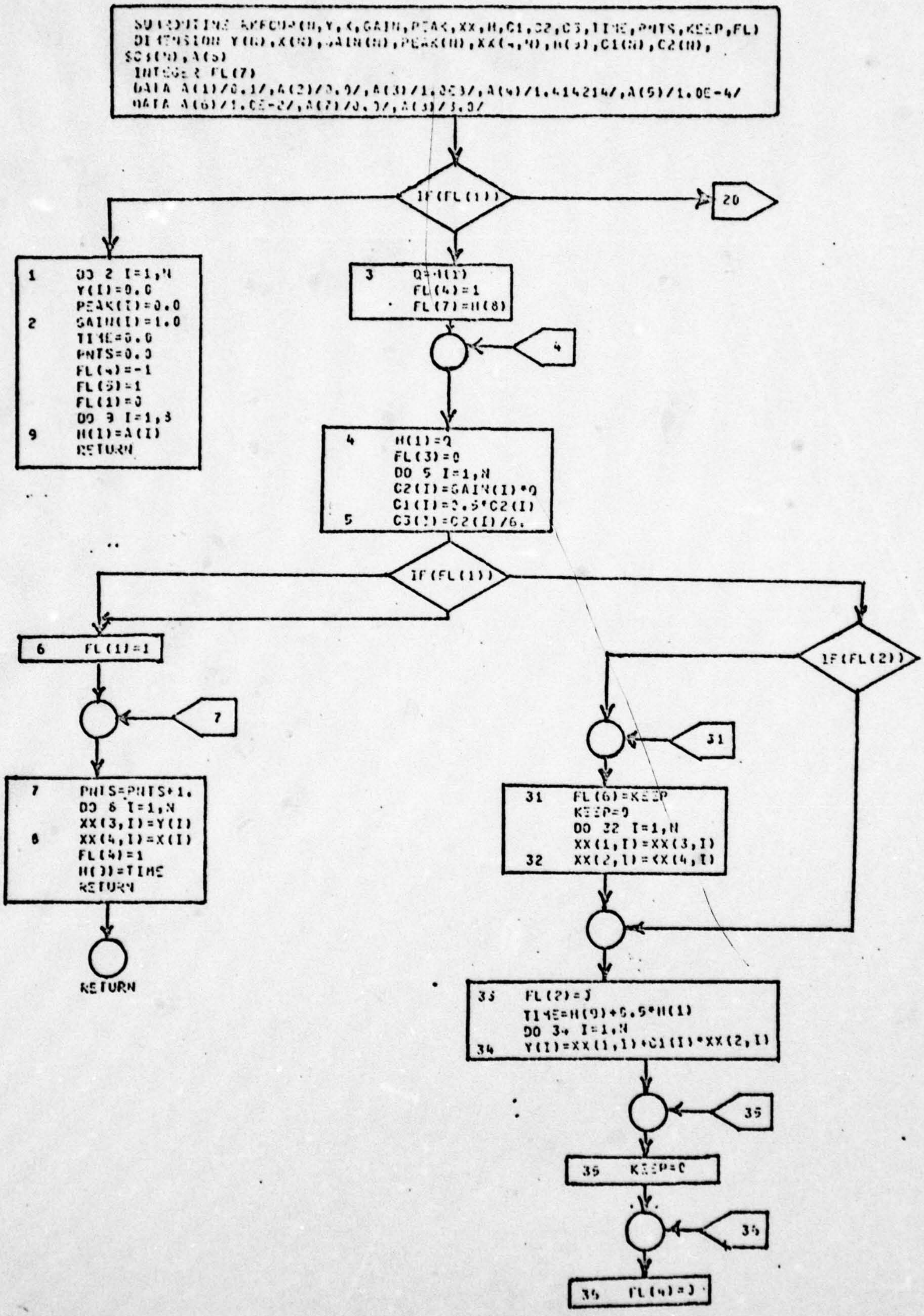
```
      DIMENSION Y(20), X(20), GA(20), PE(20), XX(4,20), H(9)
      DIMENSION ZZ1(20),ZZ2(20),ZZ3(20),IFL(7)
      DIMENSION AR(800)
      LOGICAL*1 LR(400)
      REAL IR1
C     SECTION 1 INSERTIONS *******************************
      C=1.0E-06
      R=1.0E+04
C     IPNT=0 HERE IF SUMMARY CURVES ARE DESIRED
C     NRUN=
      NRUN=1
      DO 6 IRUN=1,NRUN
      IPNT= 0
      WRITE(6,100)
      ISTOP=0
      KEEP=1
      IFL(1) = -1
C     NINT= NUMBER OF INTEGRATORS
      NINT=1
    1 IFL(2) = -1
    2 CALL RKFOUR(NINT,Y,X,GA,PE,XX,H,ZZ1,ZZ2,ZZ3,TIME,PNTS,KEEP,IFL)
      IF(IFL(4))3,4,5
    3 INIT=1
      TIME1=-1.256789
      NC=1
C     NCC=NUMBER OF POINTS KEPT
      NCC=1
C     FINTIM=FINISH TIME
      FINTIM=0.5
C     PTMAX=MAXIMUM NUMBER OF POINTS
      PTMAX=400
C     SECTION 2 INSERTIONS    *******************************
      Y(1)=-1.0
      GA(1)=1.0/C
      WRITE(6,101)
    4 IF(TIME-TIME1) 7,8,7
    7 TIME1=TIME
C     SECTION 3 INSERTIONS    *******************************
      V=1.0
    8 CONTINUE
C     SECTION 4 INSERTIONS    ************************
      VC=Y(1)
      IR1=(V-VC)/R
      X(1)=IR1
      GO TO 2
    5 IPR=0
      NC=NC-1
      IF (NC)9,9,10
    9 NC=NCC
      IPR=1
   10 IF(TIME.GE.FINTIM.OR.PNTS,GE.PTMAX)ISTOP=1
      IF((IPR+ISTOP).EQ.0) GO TO 11
C     SECTION 5 INSERTIONS     *********************************
```

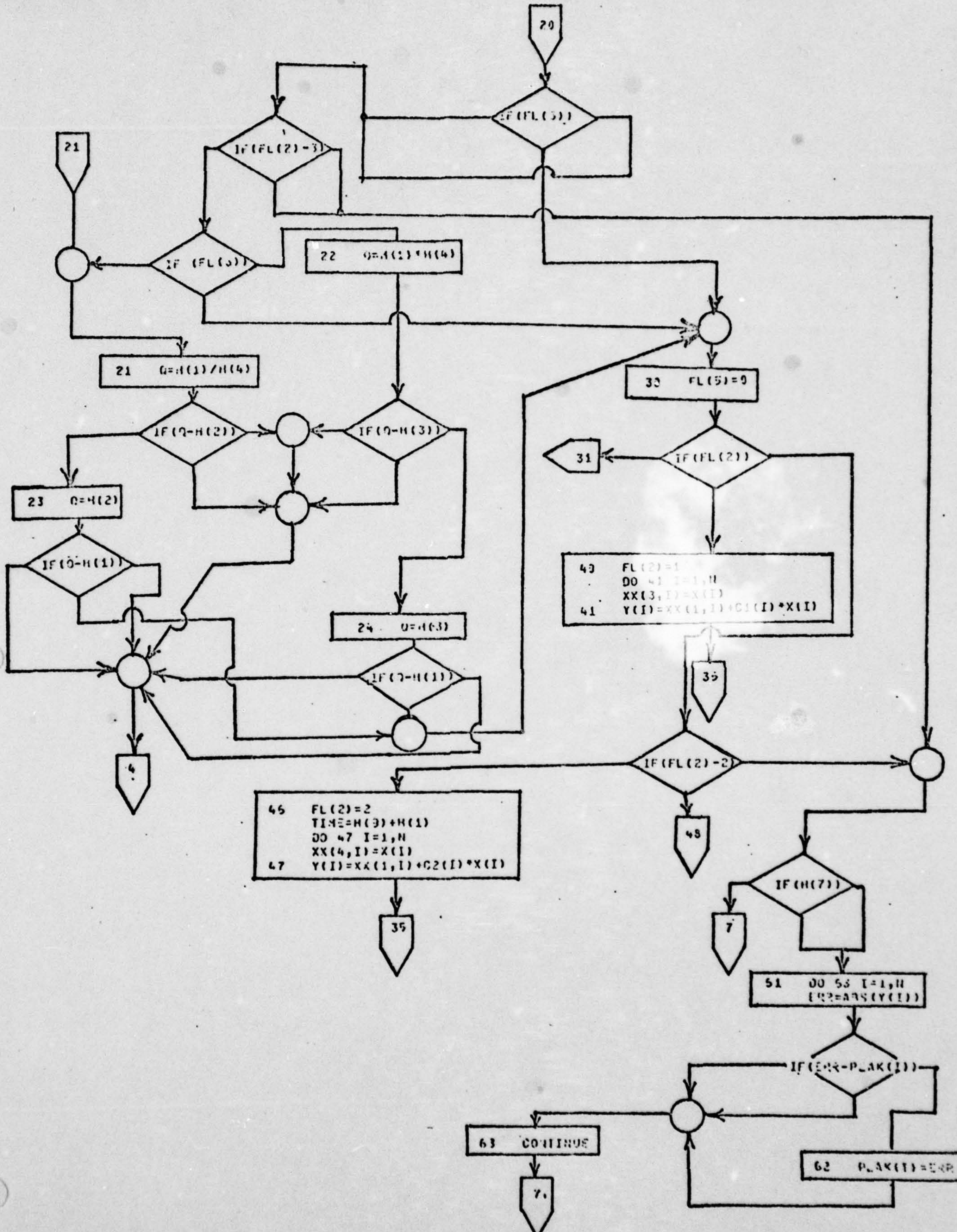
```
           WRITE(6,102) TIME,IR1,VC
           CALL FPLOT(800,IPNT,AR,LR,0,1,2,TIME,V)
           CALL FPLOT(800,IPNT,AR,LR,ISTOP,2,2,TIME,VC)
   11      INIT=0
           IF(ISTOP)1,1,6
   6       CONTINUE
           WRITE(6,100)
   100     FORMAT(1H1,1X)
   101     FORMAT(5X,*TIME*,T25,'RESISTOR CURRENT*,T50,'CAPACITOR VOLTAGE)
   102     FORMAT(2X,1PE12.5,T26,E15.7,T52,E15.7)
           STOP
           END
 /DATA
           SUBROUTINE RKFOUR(N,Y,X,GAIN,PEAK,XX,H,C1,C2,C3,TIME,PUTS,KEEP,FL)
           DIMENSION Y(N), X(N), GAIN(N), PEAK(N), XX(4,N)H(9),C1(N),C2(N),
          $C3(N),A(8)
           INTEGER FL(7)
           DATA A(1)/0.1/,A(2)/0.0/,A(3)/1.0E8/,A(4)/1.414214/,A(5)/1.03-4/
           DATA A(6)/1.0E-2/,A(7)/0.0/,A(8)/3.0/
           IF(FL(1))1,3,20
         1 DO 2 I=1,N
           Y(I)=0.0
           PEAK(I)=0.0
         2 GAIN(I)=1.0
           TIME=0.0
           PNTS=0.0
           FL(4)=-1
           FL(5)=1
           FL(1)=0
           DO 9 I=1,8
         9 H(I)=A(I)
           RETURN
         3 Q=H(1)
           FL(4)=1
           FL(7)=H(8)
         4 H(1)=Q
           FL(3)=0
           DO 5 I=1,N
           C2(I)=GAIN(I)*Q
           C1(I)=0.5*C2(I)
         5 C3(I)=C2(I)/6.
           IF(FL(1))6,6,10
        10 IF(FL(2))31,33,33
         6 FL(1)=1
         7 PNTS=PNTS+1.
           DO 8 I=1,N
           XX(3,I)=Y(I)
         8 XX(4,I)=X(I)
           FL(4)=1
           H(9)=TIME
           RETURN
        20 IF(FL(3))25,30,25
        25 IF(FL(2)-3)26,60,60
        26 IF(FL(3))21,30,22
        21 Q=H(1)/H(4)
```

```
      IF(Q-H(2))23,4,4
   23 Q=H(2)
      IF(Q-H(2))23,4,4
   22 Q=H(1)*H(4)
      IF(Q-H(3))4,4,24
   24 Q=H(3)
      IF(Q-H(1))4,30,4
   30 FL(5)=0
      IF(FL(2))31,40,45
   31 FL(6)=KEEP
```

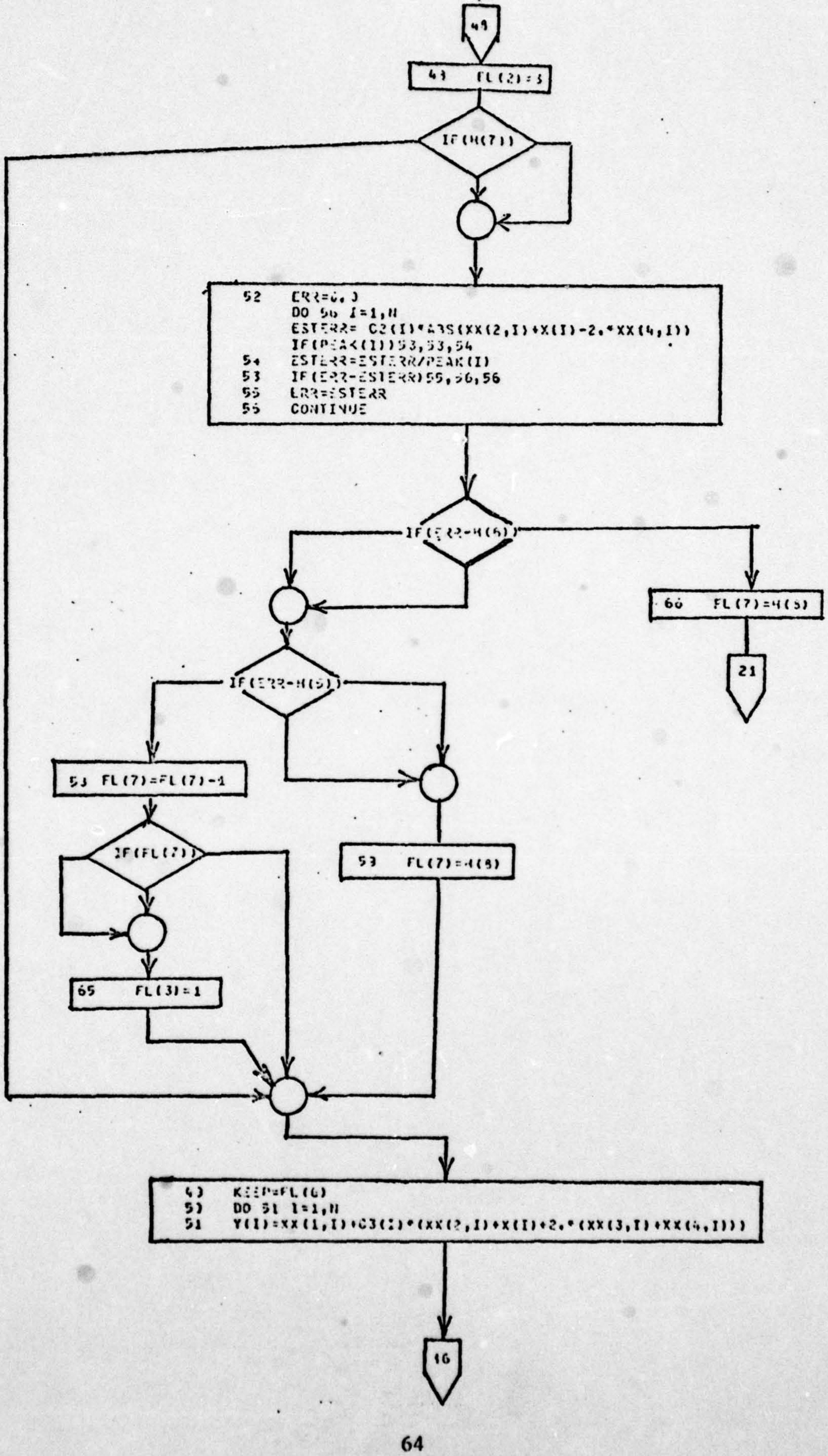
TIME	RESISTOR CURRENT	CAPACITOR VOLTAGE
0.	2.0000000E-04	-1.0000000E+00
1.00000E-03	1.8096750E-04	-8.0967500E-01
2.00000E-03	1.6374618E-04	-6.3746180E-01
3.00000E-03	1.4816368E-04	-4.8163684E-01
4.41421E-03	1.2862443E-04	-2.8624431E-01
5.82843E-03	1.1166194E-04	-1.1661938E-01
7.24264E-03	9.6936394E-05	3.0636062E-02
8.65686E-03	8.4152797E-05	1.5847203E-01
1.00711E-02	7.3055052E-05	2.6944948E-01
1.14853E-02	6.3420834E-05	3.6579166E-01
1.28995E-02	5.5057140E-05	4.4942860E-01
1.43137E-02	4.7796417E-05	5.2203583E-01
1.63137E-02	3.9132515E-05	6.0867485E-01
1.83137E-02	3.2039091E-05	6.7960909E-01
2.03137E-02	2.6231468E-05	7.3768532E-01
2.23137E-02	2.1476575E-05	7.8523425E-01
2.43137E-02	1.7583585E-05	8.2416415E-01
2.63137E-02	1.4396266E-05	8.5603734E-01
2.83137E-02	1.1786701E-05	8.8213299E-01
3.03137E-02	9.6501639E-06	9.0349836E-01
3.31422E-02	7.2728704E-06	9.2727130E-01
3.59706E-02	5.4812171E-06	9.4518783E-01
3.87990E-02	4.1309331E-06	9.5869067E-01
4.16274E-02	3.1132831E-06	9.6886712E-01
4.44559E-02	2.3463374E-06	9.7653663E-01
4.84559E-02	1.5729838E-06	9.8427016E-01
5.24559E-02	1.0545278E-06	9.8945472E-01
5.64559E-02	7.0695512E-07	9.9293045E-01
6.04559E-02	4.7394248E-07	9.9526058E-01
6.44559E-02	3.1773088E-07	9.9682269E-01
7.01128E-02	1.8060168E-07	9.9819398E-01
7.81128E-02	8.1583683E-08	9.9918416E-01
8.61128E-02	3.6854017E-08	9.9963146E-01
9.41128E-02	1.6648164E-08	9.9983352E-01
1.02113E-01	7.5205198E-09	9.9992479E-01
1.13427E-01	2.5234055E-09	9.9997477E-01
1.29427E-01	6.8232887E-10	9.9999318E-01
1.52054E-01	3.1295426E-10	9.9999687E-01
1.84054E-01	5.7200481E-10	9.9999428E-01
2.29309E-01	5.0015725E-09	9.9994998E-01
2.74564E-01	4.3733422E-08	9.9956267E-01
2.97192E-01	2.0058598E-08	9.9979941E-01
3.19819E-01	9.1999974E-09	9.9990800E-01
3.42447E-01	4.2196344E-09	9.9995780E-01
3.65074E-01	1.9353508E-09	9.9998065E-01
3.87702E-01	8.8766493E-10	9.9999112E-01
4.10329E-01	4.0713288E-10	9.9999593E-01
4.32957E-01	1.8673396E-10	9.9999813E-01
4.55584E-01	8.5646658E-11	9.9999914E-01
4.87584E-01	1.5654141E-10	9.9999843E-01
5.32839E-01	1.3687878E-09	9.9998631E-01

Flow Chart for RKFOUR





20
IF(FL(5))
IF(FL(2)-3)
21
IF (FL(5))
22 Q=H(1)*H(4)
21 Q=H(1)/H(4)
IF(Q-H(2))
IF(Q-H(3))
23 Q=H(2)
IF(Q-H(1))
24 Q=H(3)
IF(Q-H(1))
4
30 FL(5)=0
31
IF(FL(2))
40 FL(2)=1
DO 41 I=1,N
XX(3,I)=X(I)
41 Y(I)=XX(1,I)+C1(I)*X(I)
35
IF(FL(2)-2)
46 FL(2)=2
TIME=H(3)+H(1)
DO 47 I=1,N
XX(4,I)=X(I)
47 Y(I)=XX(1,I)+C2(I)*X(I)
35
48
IF(H(7))
7
51 DO 63 I=1,N
ERR=ABS(Y(I))
IF(ERR-PEAK(I))
63 CONTINUE
62 PEAK(I)=ERR
7



49
43 FL(2)=3
IF(H(7))
52 ERR=0.0
DO 56 I=1,N
ESTERR= C2(I)*ABS(XX(2,I)+X(I)-2.*XX(4,I))
IF(PEAK(I))53,53,54
54 ESTERR=ESTERR/PEAK(I)
53 IF(ERR-ESTERR)55,56,56
55 ERR=ESTERR
56 CONTINUE
IF(ERR-H(6))
66 FL(7)=H(5)
21
IF(ERR-H(5))
58 FL(7)=FL(7)-1
IF(FL(7))
59 FL(7)=H(8)
65 FL(3)=1
49 KEEP=FL(6)
50 DO 51 I=1,N
51 Y(I)=XX(1,I)+C3(I)*(XX(2,I)+X(I)+2.*(XX(3,I)+XX(4,I)))
16

Source Listing of the Simulation Program

```
LWV,T25,I070,CM60000,STCSB.T780493,VANNOY,53835
FTN,PL=8000,OPT=3,I=INPUT,L=OUTPUT,B=LGO.
LGO.
```

These job cards are needed for the digital simulator

```
                    74/74    OPT=2                              FTN 4.6+446

      PROGRAM LPN(INPUT,OUTPUT)
C
C
C*********************************************************************
C*********************************************************************
C*********************************************************************
C*********************************************************************
C
C
C  THIS PROGRAM SIMULATES THE DISCHARGE OF THE E-LINE PULSE-FORMING
C  NETWORK.  THIS IS DONE BY FIRST FORMING AN ANALOG MODEL OF THE
C  NETWORK.  THE MODEL IS THEN USED IN A COMPUTER SUBROUTINE THAT
C  APPROXIMATES AN ANALOG COMPUTER.
C  AN ANALOG MODEL OF THE E-LINE IS DEVELOPED IN CHAPTER IV OF THIS
C  THESIS.  THE COMPUTER SUBROUTINE RKFOUR APPROXIMATES AN ANALOG
C  COMPUTER.  THEREFORE, IF THE EQUASIONS USED TO DEVELOP THE ANALOG
C  MODEL ARE PROGRAMMED INTO RKFOUR THE DISCHARGING E-LINE PFN CAN BE
C  APPROXIMATED (SEE THE OPERATING INSTRUCTIONS FOR RKFOUR).
C
C  THE MAIN PROGRAM WHICH CALLS RKFOUR HAS BEEN SET UP TO HANDLE
C     1.  VARYING PULSE RATES
C     2.  VARYING NUMBERS OF MESHES
C     3.  VARYING INITIAL VOLTAGES ON THE SOURCE CAPACITOR
C  THE CALLING PROGRAM HAS ALSO BEEN PROGRAMED TO MODEL THE SWITCHING
C  THAT TAKES PLACE IN THE NETWORK AS THE NETWORK DISCHARGES.  RECALL,
C  A SWITCH BETWEEN CS AND L(1) CLOSES AT THE START OF THE PULSE,
C  OPENS WHEN THE CURRENT IN L(1) GOES NEGATIVE, CLOSES AGAIN WHEN
C  VCS BECOMES LARGER THAN VC1, AND OPENS AGAIN WHEN THE CURRENT IN
C  L(1) AGAIN GOES NEGATIVE.  THIS SWITCHING CONTINUES UNTIL VCS
C  REACHES A PREDETERMINED VALUE (SEE PAGES 26-28 OF THIS THESIS).
C
C  THE CALLING PROGRAM CONTAINS FIVE SECTIONS, A PROBLEM STATEMENT IS
C  PLACED IN A PARTICULAR SECTION DEPENDING ON THE PURPOSE OF THE
C  STATEMENT.  SEE THE OPERATING INSTRUCTIONS FOR AN EXPLAINATION OF
C  EACH SECTION.
C
C
C*********************************************************************
C*********************************************************************
C*********************************************************************
C*********************************************************************
C
C
      DIMENSION Y(20),X(20),GA(20),PE(20),XX(4,20),H(9)
      DIMENSION C(20),R(20),NFL(5)
      DIMENSION ZZ1(20),ZZ2(20),ZZ3(20),IFL(7)
      REAL IL1,IL2,IL3,IC1,IC2,IC3,IRL,L(20)
C
C*********************************************************************
C  *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
C
C        MANY OF THE VARIABLES IN THE PROGRAM ARE DEFINED IN THE
C        OPERATING INSTRUCTIONS FOR RKFOUR.  THOSE VARIABLES WHICH HAVE
C        NOT BEEN DEFINED ARE DEFINED BELOW:
C
C        RL------THE RESISTANCE OF THE LOAD RESISTOR IN OHMS.
C
```

```
C         L(J)----THE INDUCTANCE OF THE INDUCTOR IN THE J'TH MESH.
C                 (HENRYS)
C                 J=1,2,3,...,M; WHERE, M=THE NUMBER OF MESHES.
C
C         C(J)----THE CAPACITANCE OF THE CAPACITOR IN THE J'TH MESH.
C                 (FARADS)
C
C         R(J)----THE RESISTANCE OF THE RESISTOR IN THE J'TH MESH.
C                 (OHMS)
C
C         PULSEC--THE NUMBER OF PULSES PER SECOND
C
C         TSTEP---THE TIME BETWEEN THE BEGINNING OF ONE PULSE AND THE
C                 BEGINNING OF THE NEXT.  (SECONDS)
C                 =1/PULSEC
C
C         NFL(K)--USED IN THE PREPULSE TIME STEPPER (SEE SECTION 3)
C                 K=1,2,3
C
C         TIMEL10-USED IN THE PREPULSE TIME STEPPER (SEE SECTION 3)
C
C         TIMES---USED IN THE PREPULSE TIME STEPPER (SEE SECTION 3)
C
C         XMIN----USED IN THE PREPULSE TIME STEPPER (SEE SECTION 3)
C
C         ITRIG---USED IN SECTION 4 WHEN MODELING THE SWITCH IN THE
C                 NETWORK
C
C         ZVCS----USED IN SECTION 4 WHEN MODELING THE SWITCH IN THE
C                 NETWORK
C
C  *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
C*************************************************************************
C
C
C
C
C  *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
C         *****  SECTION 1 INSERTIONS  *****
C         THIS SECTION IS ENTERED ONCE PER PROGRAM.
C  *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
C
C
C         PARAMETER ESTIMATOR L AND C VALUES
C
      RL = .165
      CS = 1.333E-04
      L(1) = 2.150E-07
      L(2) = 5.244E-08
      L(3) = 1.516E-07
      L(4) = 1.639E-09
      L(5) = 1.270E-07
      C(1) = 7.826E-06
      C(2) = 7.779E-06
      C(3) = 8.515E-06
      C(4) = 1.079E-05
      C(5) = 2.029E-05
```

```
                R(1) = 15.0E-03
                R(2) = 15.0E-03
                R(3) = 15.0E-03
                R(4) = 15.0E-03
                R(5) = 15.0E-03
      C
                PULSEC = 100.0
                TSTEP = 1.0/PULSEC
                NFL(1)=0
                NFL(2)=0
                NFL(3)=0
                TIMEL10=0.0
                TIMES=TSTEP
                ITRIG = 1
                ZVCS = 40.0E-06
      C
                XMIN = 5.0E-06
      C        THE NUMBER OF RUNS
                NRUN=1
                DO 6 IRUN=1,NRUN
                IPNT=0
                PRINT 100
                ISTOP=0
                KEEP=1
                IFL(1)=-1
      C        NINT IS THE NUMBER OF INTEGRATORS
                NINT = 11
              1 IFL(2)=-1
      C
      C
      C        CALLING THE SUBROUTINE RKFOUR
              2 CALL RKFOUR(NINT,Y,X,GA,PE,XX,H,ZZ1,ZZ2,ZZ3,TIME,PNTS,KEEP,IFL)
      C
                IF(IFL(4))3,4,5
              3 INIT=1
                TIME1 = -1.256789
                NC=1
      C        NCC IS THE NUMBER OF POINTS KEPT
                NCC = 1
      C        FINTIM IS THE FINISH TIME
                FINTIM = 0.5
      C        PTMAX IS THE MAXIMUM NUMBER OF POINTS
                PTMAX = 2000
      C
      C *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      C            *****  SECTION 2 INSERTIONS  *****
      C            THIS SECTION IS ENTERED ONCE PER RUN.
      C *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      C
                H(1) = 0.5E-06
                CSIC = 30.0E+03
                Y(1) = CSIC
                GA(1) = -1.0/CS
                GA(2) = 1.0/L(1)
                GA(3) = 1.0/C(1)
                GA(4) = 1.0/L(2)
                GA(5) = 1.0/C(2)
```

```
      GA(6) = 1.0/L(3)
      GA(7) = 1.0/C(3)
      GA(8) = 1.0/L(4)
      GA(9) = 1.0/C(4)
      GA(10) = 1.0/L(5)
      GA(11) = 1.0/C(5)
      PRINT 101
    4 IF(TIME-TIME1)7,8,7
    7 TIME1=TIME
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * *
C        ***** SECTION 3 INSERTIONS *****
C        THIS SECTION IS ENTERED TWICE PER TIME STEP.
C        A PREPULSE TIME STEPPER HAS BEEN PROGRAMED HERE. THE PREPULSE
C        STEPPER MAKES SURE THAT THE DERITIVES ARE ACCURATE JUST PRIOR
C        TO THE NEXT PULSE AND THAT THE OUTPUT VOLTAGE AND CURRENT ARE
C        CALCULATED EXACTLY AT THE BEGINNING OF A NEW PULSE. THIS IS
C        DONE BY FORCING THE SUBROUTINE TO MADE TWO CALCULATIONS JUST
C        PRIOR TO A NEW PULSE AND MAKE ONE CALCULLATION AT THE TIME OF
C        A NEW PULSE.
C * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      IF(NFL(2).EQ.2) GO TO 16
      IF(NFL(1).EQ.2) GO TO 15
      IF(TIMES-TIME.LT.XMIN) GO TO 24
      GO TO 25
   24 IF(TIME.EQ.TIMES) GO TO 16
      TIME = TIMES-ABS(TIMES-TIME)/2.0
      NFL(1) = NFL(1)+1
      GO TO 25
   15 TIME = TIMES-(1.0E-08)
      NFL(2) = NFL(2)+1
      GO TO 25
   16 TIME = TIMES
      NFL(3) = NFL(3)+1
      IF(NFL(3).LT.2) GO TO 25
      NFL(1) = 0
      NFL(2) = 0
      NFL(3) = 0
      TIMES = TIMES+TSTEP
   25 CONTINUE
   29 CONTINUE
C
    8 CONTINUE
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * *
C        ***** SECTION 4 INSERTIONS *****
C        THIS SECTION IS ENTERED 4 TIMES PER TIME STEP.
C        THE SWITCHING THAT OCCURS IN THE NETWORK IS MODELED IN THIS
C        SECTION.
C * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      VC1 = Y(3)
      IL2 = Y(4)
      VC2 = Y(5)
      IL3 = Y(6)
      VC3 = Y(7)
```

```
      IL4 = Y(8)
      VC4 = Y(9)
      IL5 = Y(10)
      VC5 = Y(11)
C
C       THIS IS THE SWITCH MODEL
C
C
      TIMEL10 = TIMES-TSTEP
      IF(TIMEL10+TIME.LT.ZVCS) GO TO 45
      GO TO 46
   45 ITRIG = 1
      GO TO 47
   46 ITRIG = 0
   47 CONTINUE
      IF(IL1.LE.0.0.AND.TIME.GT.TIMEL10+2.0E-06) GO TO 30
      GO TO 31
   30 IF(VC1.GT.VCS.AND.ITRIG.EQ.1) GO TO 32
      IF(ITRIG.EQ.0) GO TO 33
   31 VCS = Y(1)
      IL1 = Y(2)
      X(1) = IL1
      GO TO 34
   32 Y(2) = 0.0
      IL1 = Y(2)
      X(1) = IL1
      GO TO 34
   33 Y(1) = CSIC
      VCS = 0.0
      Y(2) = 0.0
      IL1 = 0.0
      X(1) = IL1
   34 CONTINUE
C
C
        VR1 = Y(2)*R(1)
      VL1 = Y(1)-Y(3)-VR1
      X(2) = VL1
      IC1 = Y(2)-Y(4)
      X(3) = IC1
        VR2 = Y(4)*R(2)
      VL2 = Y(3)-Y(5)-VR2
      X(4) = VL2
      IC2 = Y(4)-Y(6)
      X(5) = IC2
       VR3 = Y(6)*R(3)
      VL3 = Y(5)-Y(7)-VR3
      X(6) = VL3
      IC3 = Y(6)-Y(8)
      X(7) = IC3
       VR4 = Y(8)*R(4)
      VL4 = Y(7)-Y(9)-VR4
      X(8) = VL4
      IC4 = Y(8)-Y(10)
      X(9) = IC4
       VR5 = Y(10)*R(5)
      VL5 = Y(9)-Y(11)-VR5
```

```
      X(10) = VL5
      IC5 = Y(10)-(Y(11)/RL)
      X(11) = IC5
      IRL = Y(11)/RL
      GO TO 2
    5 IPR=0
      NC = NC-1
      IF(NC)9,9,10
    9 NC = NCC
      IPR=1
   10 IF(TIME.GE.FINTIM.OR.PNTS.GE.PTMAX)ISTOP=1
      IF((IPR+ISTOP).EQ.0) GO TO 11
C
C * * * * * * * * * * * * * * * * * * * * * * * * * *
C      *****  SECTION 5 INSERTIONS  *****
C       THIS SECTION CONTAINS THE OUTPUT QUANTITIES.
C * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      PRINT 102,TIME,VCS,IL1,VC1,VC5,IRL
   11 INIT=0
      IF(ISTOP)1,1,6
    6 CONTINUE
  100 FORMAT(1H1)
  101 FORMAT(6X,*TIME*,13X,*VCS*,14X,*IL1*,14X,*VC1*,14X,*VC5*,14X,
     $*IRL*)
  102 FORMAT(2X,1PE12.5,5X,1PE12.5,5X,1PE12.5,5X,1PE12.5,5X,1PE12.5,
     $5X,1PE12.5)
      STOP
      END
```

```
      SUBROUTINE RKFOUR(N,Y,X,GAIN,PEAK,XX,H,C1,C2,C3,TIME,PNTS,KEEP,FL)
      DIMENSION Y(N),X(N),GAIN(N),PEAK(N),XX(4,N),H(9),C1(N),C2(N),
     $C3(N),A(8)
      INTEGER FL(7)
      DATA A(1)/0.1/,A(2)/0.0/,A(3)/1.0E8/,A(4)/1.414214/,A(5)/1.0E-4/
      DATA A(6)/1.0E-2/,A(7)/0.0/,A(8)/3.0/
      IF(FL(1))1,3,20
    1 DO 2 I=1,N
      Y(I)=0.0
      PEAK(I)=0.0
    2 GAIN(I)=1.0
      TIME=0.0
      PNTS=0.0
      FL(4)=-1
      FL(5)=1
      FL(1)=0
      DO 9 I=1,8
    9 H(I)=A(I)
      RETURN
    3 Q=H(1)
      FL(4)=1
      FL(7)=H(8)
    4 H(1)=Q
      FL(3)=0
      DO 5 I=1,N
      C2(I)=GAIN(I)*Q
      C1(I)=0.5*C2(I)
    5 C3(I)=C2(I)/6.
      IF(FL(1))6,6,10
   10 IF(FL(2))31,33,33
    6 FL(1)=1
    7 PNTS=PNTS+1.
      DO 8 I=1,N
      XX(3,I)=Y(I)
    8 XX(4,I)=X(I)
      FL(4)=1
      H(9)=TIME
      RETURN
   20 IF(FL(3))25,30,25
   25 IF(FL(2)-3)26,60,60
   26 IF (FL(3))21,30,22
   21 Q=H(1)/H(4)
      IF(Q-H(2))23,4,4
   23 Q=H(2)
      IF(Q-H(1))4,30,4
   22 Q=H(1)*H(4)
      IF(Q-H(3))4,4,24
   24 Q=H(3)
      IF(Q-H(1))4,30,4
   30 FL(5)=0
      IF(FL(2))31,40,45
   31 FL(6)=KEEP
      KEEP=0
      DO 32 I=1,N
      XX(1,I)=XX(3,I)
   32 XX(2,I)=XX(4,I)
   33 FL(2)=0
```

```
      TIME=H(9)+0.5*H(1)
      DO 34 I=1,N
34    Y(I)=XX(1,I)+C1(I)*XX(2,I)
35    KEEP=0
36    FL(4)=0
      RETURN
40    FL(2)=1
      DO 41 I=1,N
      XX(3,I)=X(I)
41    Y(I)=XX(1,I)+C1(I)*X(I)
      GO TO 35
45    IF(FL(2)-2)46,48,60
46    FL(2)=2
      TIME=H(9)+H(1)
      DO 47 I=1,N
      XX(4,I)=X(I)
47    Y(I)=XX(1,I)+C2(I)*X(I)
      GO TO 35
48    FL(2)=3
      IF(H(7))49,52,52
49    KEEP=FL(6)
50    DO 51 I=1,N
51    Y(I)=XX(1,I)+C3(I)*(XX(2,I)+X(I)+2.*(XX(3,I)+XX(4,I)))
      GO TO 36
52    ERR=0.0
      DO 56 I=1,N
      ESTERR= C2(I)*ABS(XX(2,I)+X(I)-2.*XX(4,I))
      IF(PEAK(I))53,53,54
54    ESTERR=ESTERR/PEAK(I)
53    IF(ERR-ESTERR)55,56,56
55    ERR=ESTERR
56    CONTINUE
      IF(ERR-H(6))57,57,66
57    IF(ERR-H(5))58,59,59
58    FL(7)=FL(7)-1
      IF(FL(7))65,65,49
65    FL(3)=1
      GO TO 49
59    FL(7)=H(8)
      GO TO 49
66    FL(7)=H(8)
      GO TO 21
60    IF(H(7))7,61,61
61    DO 63 I=1,N
      ERR=ABS(Y(I))
      IF(ERR-PEAK(I))63,63,62
62    PEAK(I)=ERR
63    CONTINUE
      GO TO 7
      END
```

SYMBOLIC REFERENCE MAP (R=1)

Typical Output from the Simulation Program

TIME	VCS	IL1	VC1	VC5	IRL
0.	3.00000E+04	0.	0.	0.	0.
2.20970E-08	2.99997E+04	3.06077E+03	4.35014E+00	0.	0.
2.60033E-08	2.99996E+04	3.62482E+03	6.02325E+00	0.	0.
2.69798E-08	2.99996E+04	3.76080E+03	6.48392E+00	0.	0.
2.79564E-08	2.99996E+04	3.89677E+03	6.96154E+00	0.	0.
2.89329E-08	2.99996E+04	4.03273E+03	7.45612E+00	0.	0.
2.99095E-08	2.99995E+04	4.16867E+03	7.96764E+00	0.	0.
3.08860E-08	2.99995E+04	4.30461E+03	8.49610E+00	0.	0.
3.18626E-08	2.99995E+04	4.44053E+03	9.04151E+00	0.	0.
3.28392E-08	2.99994E+04	4.57645E+03	9.60386E+00	0.	0.
3.38157E-08	2.99994E+04	4.71235E+03	1.01831E+01	0.	0.
3.47923E-08	2.99994E+04	4.84323E+03	1.07794E+01	0.	0.
3.57688E-08	2.99993E+04	4.98411E+03	1.13925E+01	0.	0.
3.67454E-08	2.99993E+04	5.11997E+03	1.20226E+01	0.	0.
3.77219E-08	2.99993E+04	5.25582E+03	1.26696E+01	0.	0.
3.86985E-08	2.99992E+04	5.39166E+03	1.33335E+01	0.	0.
3.96751E-08	2.99992E+04	5.52749E+03	1.40144E+01	0.	0.
4.06516E-08	2.99991E+04	5.66330E+03	1.47121E+01	0.	0.
4.16282E-08	2.99991E+04	5.79910E+03	1.54268E+01	0.	0.
4.26047E-08	2.99991E+04	5.93489E+03	1.61584E+01	0.	0.
4.35813E-08	2.99990E+04	6.07066E+03	1.69068E+01	0.	0.
4.49623E-08	2.99989E+04	6.26266E+03	1.79942E+01	0.	0.
4.63434E-08	2.99989E+04	6.45453E+03	1.91154E+01	0.	0.
4.77245E-08	2.99988E+04	6.64657E+03	2.02703E+01	0.	0.
4.91055E-08	2.99987E+04	6.83848E+03	2.14591E+01	0.	0.
5.04866E-08	2.99987E+04	7.03037E+03	2.26816E+01	0.	0.
5.18676E-08	2.99986E+04	7.22223E+03	2.39378E+01	0.	0.
5.32487E-08	2.99985E+04	7.41406E+03	2.52278E+01	0.	0.
5.46298E-08	2.99984E+04	7.60587E+03	2.65515E+01	0.	0.
5.60108E-08	2.99984E+04	7.79765E+03	2.79090E+01	0.	0.
5.73919E-08	2.99983E+04	7.98940E+03	2.93002E+01	0.	0.
5.93450E-08	2.99982E+04	8.26053E+03	3.13251E+01	0.	0.
6.12981E-08	2.99980E+04	8.53160E+03	3.34174E+01	0.	0.
6.32512E-08	2.99979E+04	8.80262E+03	3.55770E+01	0.	0.
6.52044E-08	2.99978E+04	9.07357E+03	3.78040E+01	0.	0.
6.71575E-08	2.99976E+04	9.34447E+03	4.00982E+01	0.	0.
6.91106E-08	2.99975E+04	9.61531E+03	4.24596E+01	0.	0.
7.10637E-08	2.99974E+04	9.88609E+03	4.48882E+01	0.	0.
7.30168E-08	2.99972E+04	1.01568E+04	4.73840E+01	0.	0.
7.49699E-08	2.99971E+04	1.04275E+04	4.99469E+01	0.	0.
7.69230E-08	2.99969E+04	1.06981E+04	5.25769E+01	0.	0.
7.88762E-08	2.99963E+04	1.09686E+04	5.52739E+01	0.	0.
8.08293E-08	2.99966E+04	1.12391E+04	5.80379E+01	0.	0.
8.35914E-08	2.99964E+04	1.16215E+04	6.20611E+01	0.	0.
8.63535E-08	2.99961E+04	1.20037E+04	6.52181E+01	0.	0.
8.91156E-08	2.99959E+04	1.23859E+04	7.05088E+01	0.	0.
9.18778E-08	2.99956E+04	1.27679E+04	7.49330E+01	0.	0.
9.46399E-08	2.99953E+04	1.31497E+04	7.94907E+01	0.	0.
9.74020E-08	2.99950E+04	1.35314E+04	8.41816E+01	0.	0.
1.00164E-07	2.99948E+04	1.39130E+04	8.90057E+01	0.	0.
1.02926E-07	2.99945E+04	1.42945E+04	9.39628E+01	0.	0.
1.05689E-07	2.99942E+04	1.46753E+04	9.90528E+01	0.	0.
1.08451E-07	2.99939E+04	1.50570E+04	1.04276E+02	0.	0.
1.11213E-07	2.99935E+04	1.54330E+04	1.09631E+02	0.	0.
1.15119E-07	2.99931E+04	1.59766E+04	1.17430E+02	0.	0.
1.19025E-07	2.99926E+04	1.65149E+04	1.25495E+02	0.	0.
1.22931E-07	2.99921E+04	1.70529E+04	1.33823E+02	0.	0.
[illegible]	2.99916E+04	1.75905E+04	1.42414E+02	0.	0.
[illegible]	2.99911E+04	1.81279E+04	1.51269E+02	0.	0.
[illegible]	2.99905E+04	1.86649E+04	1.60386E+02	0.	0.
[illegible]	2.99900E+04	1.92017E+04	1.69765E+02	0.	0.
[illegible]	2.99894E+04	1.97381E+04	1.79407E+02	0.	0.
[illegible]	[illegible]	[illegible]	[illegible]	0.	0.
⋮	⋮	⋮	⋮	⋮	⋮

⋮	⋮	⋮	⋮	⋮	⋮
6.91507E-07	2.97596E+04	8.97824E+04	3.73339E+03	0.	0.
6.99319E-07	2.97543E+04	9.06775E+04	3.80779E+03	0.	0.
7.07132E-07	2.97490E+04	9.15692E+04	3.88260E+03	0.	0.
7.14944E-07	2.97436E+04	9.24575E+04	3.95782E+03	0.	0.
7.22757E-07	2.97381E+04	9.33424E+04	4.03344E+03	0.	0.
7.30569E-07	2.97326E+04	9.42238E+04	4.10943E+03	0.	0.
7.38382E-07	2.97271E+04	9.51018E+04	4.18581E+03	0.	0.
7.46194E-07	2.97215E+04	9.59763E+04	4.26254E+03	3.20867E-04	1.94465E-03
7.54007E-07	2.97158E+04	9.68474E+04	4.33962E+03	7.05908E-04	4.27823E-03
7.61819E-07	2.97101E+04	9.77149E+04	4.41704E+03	1.09095E-03	6.61181E-03
7.69632E-07	2.97044E+04	9.85790E+04	4.49479E+03	1.47599E-03	8.94538E-03
7.77444E-07	2.96986E+04	9.94396E+04	4.57286E+03	1.86103E-03	1.12790E-02
7.85257E-07	2.96927E+04	1.00297E+05	4.65123E+03	2.24607E-03	1.36125E-02
7.93069E-07	2.96868E+04	1.01150E+05	4.72990E+03	2.63111E-03	1.59461E-02
8.00882E-07	2.96809E+04	1.02000E+05	4.80685E+03	3.01615E-03	1.82797E-02
8.08694E-07	2.96749E+04	1.02846E+05	4.88809E+03	3.40119E-03	2.06133E-02
8.10075E-07	2.96738E+04	1.02996E+05	4.90212E+03	3.46926E-03	2.10258E-02
8.10766E-07	2.96733E+04	1.03070E+05	4.90914E+03	3.50896E-03	2.12664E-02
8.11456E-07	2.96727E+04	1.03145E+05	4.91616E+03	3.57703E-03	2.16790E-02
8.12147E-07	2.96722E+04	1.03219E+05	4.92319E+03	3.64509E-03	2.20915E-02
8.12837E-07	2.96717E+04	1.03294E+05	4.93021E+03	3.71316E-03	2.25040E-02
8.13814E-07	2.96709E+04	1.03399E+05	4.94015E+03	3.80942E-03	2.30874E-02
8.15195E-07	2.96696E+04	1.03548E+05	4.95422E+03	3.94555E-03	2.39124E-02
8.17148E-07	2.96683E+04	1.03758E+05	4.97412E+03	4.13807E-03	2.50792E-02
8.19910E-07	2.96662E+04	1.04055E+05	5.00229E+03	4.41034E-03	2.67293E-02
8.23816E-07	2.96631E+04	1.04475E+05	5.04219E+03	4.79538E-03	2.90629E-02
8.27723E-07	2.96600E+04	1.04893E+05	5.08215E+03	5.18042E-03	3.13965E-02
8.31629E-07	2.96570E+04	1.05311E+05	5.12216E+03	5.56546E-03	3.37300E-02
8.35535E-07	2.96539E+04	1.05727E+05	5.16224E+03	5.95050E-03	3.60636E-02
8.39441E-07	2.96508E+04	1.06143E+05	5.20237E+03	6.33554E-03	3.83972E-02
8.43348E-07	2.96476E+04	1.06558E+05	5.24256E+03	6.72058E-03	4.07308E-02
8.47254E-07	2.96445E+04	1.06972E+05	5.28280E+03	7.10562E-03	4.30644E-02
8.51160E-07	2.96414E+04	1.07385E+05	5.32309E+03	7.49066E-03	4.53979E-02
8.53922E-07	2.96391E+04	1.07676E+05	5.35162E+03	7.76292E-03	4.70480E-02
8.55303E-07	2.96380E+04	1.07822E+05	5.36589E+03	7.95578E-03	4.82168E-02
8.56684E-07	2.96369E+04	1.07967E+05	5.38017E+03	8.15998E-03	4.94544E-02
8.58065E-07	2.96358E+04	1.08113E+05	5.39445E+03	8.36417E-03	5.06920E-02
8.59446E-07	2.96347E+04	1.08258E+05	5.40874E+03	8.56837E-03	5.19295E-02
8.61399E-07	2.96331E+04	1.08463E+05	5.42896E+03	8.85715E-03	5.36797E-02
8.64162E-07	2.96308E+04	1.08753E+05	5.45757E+03	9.26555E-03	5.61548E-02
8.68068E-07	2.96276E+04	1.09162E+05	5.49808E+03	9.84311E-03	5.96552E-02
8.73592E-07	2.96231E+04	1.09739E+05	5.55545E+03	1.06599E-02	6.46055E-02
8.79116E-07	2.96185E+04	1.10314E+05	5.61290E+03	1.14767E-02	6.95557E-02
8.84641E-07	2.96140E+04	1.10888E+05	5.67044E+03	1.22935E-02	7.45060E-02
8.86594E-07	2.96123E+04	1.11090E+05	5.69080E+03	1.25823E-02	7.62562E-02
8.88547E-07	2.96107E+04	1.11292E+05	5.71118E+03	1.29513E-02	7.84925E-02
8.90500E-07	2.96091E+04	1.11494E+05	5.73156E+03	1.33363E-02	8.08261E-02
8.92453E-07	2.96074E+04	1.11695E+05	5.75195E+03	1.37213E-02	8.31597E-02
8.94406E-07	2.96058E+04	1.11897E+05	5.77235E+03	1.41064E-02	8.54933E-02
8.97168E-07	2.96035E+04	1.12181E+05	5.80122E+03	1.46509E-02	8.87934E-02
9.01075E-07	2.96002E+04	1.12582E+05	5.84208E+03	1.54210E-02	9.34606E-02
9.06590E-07	2.95955E+04	1.13149E+05	5.89992E+03	1.65101E-02	1.00061E-01
9.12123E-07	2.95908E+04	1.13713E+05	5.95783E+03	1.75991E-02	1.06661E-01
9.14885E-07	2.95884E+04	1.13994E+05	5.98680E+03	1.82571E-02	1.10649E-01
9.17647E-07	2.95861E+04	1.14275E+05	6.01580E+03	1.89377E-02	1.14774E-01
9.20409E-07	2.95837E+04	1.14555E+05	6.04481E+03	1.96184E-02	1.18899E-01
9.23171E-07	2.95813E+04	1.14836E+05	6.07383E+03	2.02991E-02	1.23025E-01
9.27078E-07	2.95780E+04	1.15231E+05	6.11490E+03	2.12617E-02	1.28859E-01
9.32602E-07	2.95732E+04	1.15789E+05	6.17303E+03	2.26230E-02	1.37109E-01
9.36508E-07	2.95698E+04	1.16182E+05	6.21416E+03	2.36177E-02	1.43137E-01
9.40414E-07	2.95664E+04	1.16574E+05	6.25532E+03	2.47728E-02	1.50138E-01
9.44321E-07	2.95629E+04	1.16966E+05	6.29650E+03	2.59279E-02	1.57139E-01
9.48227E-07	2.95595E+04	1.17356E+05	6.33770E+03	2.70830E-02	1.64140E-01
9.53751E-07	2.95546E+04	1.17907E+05	6.39601E+03	2.87166E-02	1.74040E-01
9.59275E-07	2.95497E+04	1.18455E+05	6.45435E+03	3.05771E-02	1.85316E-01
1.64800E-07	2.95448E+04	1.19002E+05	6.51273E+03	3.24930E-02	1.96866E-01

9.70324E-07	2.95399E+04	1.19547E+05	6.57114E+03	3.43898E-02	2.08417E-01
9.75848E-07	2.95349E+04	1.20090E+05	6.62958E+03	3.65215E-02	2.21343E-01
9.81372E-07	2.95299E+04	1.20632E+05	6.68804E+03	3.86997E-02	2.34543E-01
9.86897E-07	2.95249E+04	1.21171E+05	6.74652E+03	4.08778E-02	2.47744E-01
9.92421E-07	2.95199E+04	1.21708E+05	6.80503E+03	4.32828E-02	2.62320E-01
9.97945E-07	2.95148E+04	1.22245E+05	6.86355E+03	4.57332E-02	2.77171E-01
1.00347E-06	2.95097E+04	1.22778E+05	6.92203E+03	4.82289E-02	2.92297E-01
1.00899E-06	2.95046E+04	1.23311E+05	6.98063E+03	5.09516E-02	3.08797E-01
1.01452E-06	2.94995E+04	1.23841E+05	7.03917E+03	5.37196E-02	3.25573E-01
1.02004E-06	2.94944E+04	1.24369E+05	7.09773E+03	5.67145E-02	3.43724E-01
1.02557E-06	2.94892E+04	1.24896E+05	7.15628E+03	5.97094E-02	3.61875E-01
1.03109E-06	2.94840E+04	1.25421E+05	7.21483E+03	6.29312E-02	3.81401E-01
1.03661E-06	2.94788E+04	1.25944E+05	7.27338E+03	6.61984E-02	4.01202E-01
1.04214E-06	2.94736E+04	1.26465E+05	7.33191E+03	6.96925E-02	4.22379E-01
1.04766E-06	2.94683E+04	1.26984E+05	7.39044E+03	7.32319E-02	4.43830E-01
1.05319E-06	2.94631E+04	1.27501E+05	7.44895E+03	7.69982E-02	4.66656E-01
1.05871E-06	2.94578E+04	1.28017E+05	7.50745E+03	8.08553E-02	4.90032E-01
1.06424E-06	2.94525E+04	1.28531E+05	7.56592E+03	8.49393E-02	5.14784E-01
1.06976E-06	2.94471E+04	1.29043E+05	7.62438E+03	8.91594E-02	5.40360E-01
1.07528E-06	2.94418E+04	1.29553E+05	7.68280E+03	9.35156E-02	5.66761E-01
1.08081E-06	2.94364E+04	1.30061E+05	7.74120E+03	9.80987E-02	5.94538E-01
1.08633E-06	2.94310E+04	1.30567E+05	7.79957E+03	1.02773E-01	6.22864E-01
1.09186E-06	2.94256E+04	1.31072E+05	7.85791E+03	1.07673E-01	6.52566E-01
1.09738E-06	2.94201E+04	1.31575E+05	7.91621E+03	1.12801E-01	6.83643E-01
1.10291E-06	2.94146E+04	1.32076E+05	7.97447E+03	1.18201E-01	7.16369E-01
1.10843E-06	2.94092E+04	1.32575E+05	8.03269E+03	1.23692E-01	7.49646E-01
1.11395E-06	2.94037E+04	1.33072E+05	8.09087E+03	1.29409E-01	7.84298E-01
1.11948E-05	2.93981E+04	1.33567E+05	8.14900E+03	1.35354E-01	8.20325E-01
1.12500E-06	2.93926E+04	1.34061E+05	8.20708E+03	1.41570E-01	8.58002E-01
1.13053E-06	2.93870E+04	1.34553E+05	8.26511E+03	1.48059E-01	8.97329E-01
1.13605E-06	2.93814E+04	1.35043E+05	8.32309E+03	1.54639E-01	9.37206E-01
1.14158E-06	2.93758E+04	1.35531E+05	8.38101E+03	1.61491E-01	9.78733E-01
1.14710E-06	2.93702E+04	1.36017E+05	8.43888E+03	1.68615E-01	1.02191E+00
1.15262E-06	2.93646E+04	1.36501E+05	8.49668E+03	1.76012E-01	1.06674E+00
1.15815E-06	2.93589E+04	1.36984E+05	8.55443E+03	1.83681E-01	1.11322E+00
1.16367E-06	2.93532E+04	1.37465E+05	8.61210E+03	1.91622E-01	1.16134E+00
1.16920E-06	2.93475E+04	1.37944E+05	8.66971E+03	1.99835E-01	1.21112E+00
1.17472E-06	2.93418E+04	1.38421E+05	8.72725E+03	2.08321E-01	1.26255E+00
1.18025E-06	2.93360E+04	1.38897E+05	8.78472E+03	2.17078E-01	1.31563E+00
1.18577E-06	2.93303E+04	1.39370E+05	8.84212E+03	2.26290E-01	1.37145E+00
1.19129E-06	2.93245E+04	1.39842E+05	8.89944E+03	2.35774E-01	1.42893E+00
1.19682E-06	2.93187E+04	1.40312E+05	8.95669E+03	2.45575E-01	1.48834E+00
1.20234E-06	2.93128E+04	1.40780E+05	9.01385E+03	2.55695E-01	1.54966E+00
1.20787E-06	2.93070E+04	1.41247E+05	9.07093E+03	2.66056E-01	1.61264E+00
1.21339E-06	2.93011E+04	1.41711E+05	9.12793E+03	2.76931E-01	1.67837E+00
1.21892E-06	2.92953E+04	1.42174E+05	9.18485E+03	2.88094E-01	1.74602E+00
1.22444E-06	2.92894E+04	1.42635E+05	9.24167E+03	2.99575E-01	1.81560E+00
1.22996E-06	2.92834E+04	1.43094E+05	9.29841E+03	3.11554E-01	1.88321E+00
1.23549E-06	2.92775E+04	1.43552E+05	9.35506E+03	3.23851E-01	1.95274E+00
1.24101E-06	2.92715E+04	1.44003E+05	9.41162E+03	3.36648E-01	2.04029E+00
1.24654E-06	2.92656E+04	1.44461E+05	9.46303E+03	3.49762E-01	2.11977E+00
1.25206E-06	2.92596E+04	1.44914E+05	9.52444E+03	3.63375E-01	2.20227E+00
1.25758E-06	2.92535E+04	1.45364E+05	9.58071E+03	3.77306E-01	2.28670E+00
1.26311E-06	2.92475E+04	1.45812E+05	9.63683E+03	3.91736E-01	2.37416E+00
1.26863E-06	2.92415E+04	1.46259E+05	9.69295E+03	4.06711E-01	2.46491E+00
1.27416E-06	2.92354E+04	1.46704E+05	9.74892E+03	4.22134E-01	2.55969E+00
1.27968E-06	2.92293E+04	1.47148E+05	9.80479E+03	4.37976E-01	2.65440E+00
1.28521E-06	2.92232E+04	1.47589E+05	9.86054E+03	4.54312E-01	2.75340E+00
1.29073E-06	2.92171E+04	1.48029E+05	9.91619E+03	4.71192E-01	2.85571E+00
1.29854E-06	2.92084E+04	1.48648E+05	9.99471E+03	4.95899E-01	3.00545E+00
1.30636E-06	2.91998E+04	1.49263E+05	1.00730E+04	5.21761E-01	3.16219E+00
1.31417E-06	2.91909E+04	1.49875E+05	1.01511E+04	5.46778E-01	3.32593E+00
1.32198E-06	2.91821E+04	1.50483E+05	1.02289E+04	5.76950E-01	3.49667E+00
1.32979E-06	2.91732E+04	1.51088E+05	1.03066E+04	6.06277E-01	3.67441E+00
1.33761E-06	2.91644E+04	1.51690E+05	1.03839E+04	6.37016E-01	3.86070E+00
1.34542E-06	2.91555E+04	1.52289E+05	1.04611E+04	6.68974E-01	4.05639E+00

1.35323E-06	2.91465E+04	1.52882E+05	1.05380E+04	7.02409E-01	4.25702E+00
1.36104E-06	2.91375E+04	1.53474E+05	1.06147E+04	7.37062E-01	4.46705E+00
1.36886E-06	2.91285E+04	1.54061E+05	1.06911E+04	7.73256E-01	4.68640E+00
1.37667E-06	2.91195E+04	1.54646E+05	1.07673E+04	8.10990E-01	4.91509E+00
1.38772E-06	2.91066E+04	1.55466E+05	1.08746E+04	8.67077E-01	5.25501E+00
1.40334E-06	2.90883E+04	1.56615E+05	1.10255E+04	9.51786E-01	5.76840E+00
1.42544E-06	2.90622E+04	1.58216E+05	1.12371E+04	1.06302E+00	6.55374E+00
1.44754E-06	2.90359E+04	1.59791E+05	1.14466E+04	1.22950E+00	7.45149E+00
1.46963E-06	2.90093E+04	1.61339E+05	1.16540E+04	1.39249E+00	8.43935E+00
1.49173E-06	2.89824E+04	1.62861E+05	1.18593E+04	1.57346E+00	9.53611E+00
1.51383E-06	2.89553E+04	1.64357E+05	1.20624E+04	1.77384E+00	1.07506E+01
1.53592E-06	2.89279E+04	1.65827E+05	1.22634E+04	1.99511E+00	1.20915E+01
1.55802E-06	2.89003E+04	1.67271E+05	1.24622E+04	2.23924E+00	1.35711E+01
1.58012E-06	2.88725E+04	1.68690E+05	1.26589E+04	2.50823E+00	1.52014E+01
1.60221E-06	2.88444E+04	1.70084E+05	1.28535E+04	2.80446E+00	1.69967E+01
1.62431E-06	2.88161E+04	1.71453E+05	1.30459E+04	3.12972E+00	1.89680E+01
1.65556E-06	2.87756E+04	1.73346E+05	1.33146E+04	3.64337E+00	2.20810E+01
1.68681E-06	2.87348E+04	1.75191E+05	1.35793E+04	4.22658E+00	2.56156E+01
1.71806E-06	2.86935E+04	1.76988E+05	1.38401E+04	4.88679E+00	2.96169E+01
1.74931E-06	2.86518E+04	1.78738E+05	1.40972E+04	5.63172E+00	3.41316E+01
1.79351E-06	2.85922E+04	1.81132E+05	1.44549E+04	6.84711E+00	4.14976E+01
1.85600E-06	2.85065E+04	1.84360E+05	1.49498E+04	8.93660E+00	5.41612E+01
1.91850E-06	2.84193E+04	1.87407E+05	1.54337E+04	1.15369E+01	6.99145E+01
1.98100E-06	2.83308E+04	1.90276E+05	1.59085E+04	1.47410E+01	8.93392E+01
2.04350E-06	2.82409E+04	1.92970E+05	1.63763E+04	1.86571E+01	1.13073E+02
2.10600E-06	2.81498E+04	1.95492E+05	1.68391E+04	2.34034E+01	1.41839E+02
2.16850E-06	2.80576E+04	1.97842E+05	1.72990E+04	2.91128E+01	1.76441E+02
2.23100E-06	2.79643E+04	2.00022E+05	1.77577E+04	3.59275E+01	2.17742E+02
2.29350E-06	2.78701E+04	2.02032E+05	1.82170E+04	4.40072E+01	2.66710E+02
2.35600E-06	2.77749E+04	2.03872E+05	1.86782E+04	5.35223E+01	3.24378E+02
2.41850E-06	2.76789E+04	2.05543E+05	1.91424E+04	6.46587E+01	3.91871E+02
2.48100E-06	2.75822E+04	2.07043E+05	1.96103E+04	7.76161E+01	4.70401E+02
2.54350E-06	2.74848E+04	2.08372E+05	2.00822E+04	9.26055E+01	5.61246E+02
2.60600E-06	2.73868E+04	2.09529E+05	2.05579E+04	1.09853E+02	6.65778E+02
2.69439E-06	2.72474E+04	2.10872E+05	2.12363E+04	1.38565E+02	8.39790E+02
2.81939E-06	2.70490E+04	2.12184E+05	2.22010E+04	1.89090E+02	1.14600E+03
2.94439E-06	2.68497E+04	2.12811E+05	2.31591E+04	2.53195E+02	1.53451E+03
3.06939E-06	2.66501E+04	2.12770E+05	2.40912E+04	3.33172E+02	2.01923E+03
3.19439E-06	2.64508E+04	2.12087E+05	2.49740E+04	4.31416E+02	2.61464E+03
3.31939E-06	2.62525E+04	2.10805E+05	2.57822E+04	5.50366E+02	3.33555E+03
3.44439E-06	2.60557E+04	2.08979E+05	2.64901E+04	6.92458E+02	4.19672E+03
3.56939E-06	2.58607E+04	2.06680E+05	2.70746E+04	8.60074E+02	5.21257E+03
3.69439E-06	2.56682E+04	2.03992E+05	2.75163E+04	1.05546E+03	6.39584E+03
3.81939E-06	2.54783E+04	2.01005E+05	2.78019E+04	1.28077E+03	7.76223E+03
3.94439E-06	2.52912E+04	1.97817E+05	2.79250E+04	1.53782E+03	9.32011E+03
4.06939E-06	2.51073E+04	1.94524E+05	2.78366E+04	1.82823E+03	1.10802E+04
4.24617E-06	2.48524E+04	1.89872E+05	2.75756E+04	2.29834E+03	1.39293E+04
4.42294E-06	2.46036E+04	1.85434E+05	2.70091E+04	2.84014E+03	1.72130E+04
4.59972E-06	2.43604E+04	1.81394E+05	2.62571E+04	3.45455E+03	2.09367E+04
4.77650E-06	2.41222E+04	1.77869E+05	2.54010E+04	4.14081E+03	2.50958E+04
4.95327E-06	2.38884E+04	1.74908E+05	2.45228E+04	4.89650E+03	2.96758E+04
5.13005E-06	2.36581E+04	1.72496E+05	2.36946E+04	5.71751E+03	3.46516E+04
5.30683E-06	2.34307E+04	1.70563E+05	2.29703E+04	6.59810E+03	3.99885E+04
5.48360E-06	2.32055E+04	1.69007E+05	2.23801E+04	7.53102E+03	4.56425E+04
5.66038E-06	2.29823E+04	1.67712E+05	2.19301E+04	8.50760E+03	5.15612E+04
5.83716E-06	2.27606E+04	1.66566E+05	2.16043E+04	9.51793E+03	5.76844E+04
6.01393E-06	2.25405E+04	1.65480E+05	2.13714E+04	1.05510E+04	6.39455E+04
6.19071E-06	2.23217E+04	1.64393E+05	2.11923E+04	1.15950E+04	7.02725E+04
6.36749E-06	2.21049E+04	1.63278E+05	2.10288E+04	1.26373E+04	7.65893E+04
6.54426E-06	2.18887E+04	1.62139E+05	2.08509E+04	1.36652E+04	8.28196E+04
6.72104E-06	2.16744E+04	1.60995E+05	2.06423E+04	1.46697E+04	8.88833E+04
6.89781E-06	2.14617E+04	1.59874E+05	2.04028E+04	1.56261E+04	9.47039E+04
7.07459E-06	2.12504E+04	1.58797E+05	2.01471E+04	1.65343E+04	1.00208E+05
7.25137E-06	2.10405E+04	1.57769E+05	1.99013E+04	1.73790E+04	1.05327E+05
7.42814E-06	2.08319E+04	1.56768E+05	1.96970E+04	1.81500E+04	1.10000E+05
.60492E-06	2.06247E+04	1.55749E+05	1.95645E+04	1.88389E+04	1.14174E+05

7.78170E-06	2.04189E+04	1.54644E+05	1.95266E+04	1.94387E+04	1.17810E+05
7.95847E-06	2.02146E+04	1.53374E+05	1.95935E+04	1.99445E+04	1.20376E+05
8.13525E-06	2.00122E+04	1.51856E+05	1.97604E+04	2.03533E+04	1.23354E+05
8.31203E-06	1.98120E+04	1.50021E+05	2.00075E+04	2.06645E+04	1.25239E+05
8.48880E-06	1.96144E+04	1.47823E+05	2.03027E+04	2.08791E+04	1.26540E+05
8.66558E-06	1.94201E+04	1.45243E+05	2.06065E+04	2.10005E+04	1.27276E+05
8.84236E-06	1.92294E+04	1.42301E+05	2.08778E+04	2.10338E+04	1.27478E+05
9.01913E-06	1.90428E+04	1.39044E+05	2.10306E+04	2.09858E+04	1.27186E+05
9.19591E-06	1.88607E+04	1.35548E+05	2.11889E+04	2.08645E+04	1.26451E+05
9.37269E-06	1.86833E+04	1.31903E+05	2.11905E+04	2.06792E+04	1.25328E+05
9.54946E-06	1.85109E+04	1.28202E+05	2.10882E+04	2.04400E+04	1.23879E+05
9.72624E-06	1.83433E+04	1.24528E+05	2.08969E+04	2.01576E+04	1.22167E+05
9.90302E-06	1.81805E+04	1.20945E+05	2.06504E+04	1.98428E+04	1.20259E+05
1.00798E-05	1.80224E+04	1.17491E+05	2.03764E+04	1.95065E+04	1.18221E+05
1.02566E-05	1.78688E+04	1.14174E+05	2.01115E+04	1.91592E+04	1.16116E+05
1.04333E-05	1.77196E+04	1.10977E+05	1.98854E+04	1.88110E+04	1.14006E+05
1.06101E-05	1.75745E+04	1.07859E+05	1.97196E+04	1.84710E+04	1.11946E+05
1.07869E-05	1.74335E+04	1.04770E+05	1.96238E+04	1.81477E+04	1.09986E+05
1.09637E-05	1.72966E+04	1.01655E+05	1.95965E+04	1.78479E+04	1.08169E+05
1.11405E-05	1.71639E+04	9.84665E+04	1.96255E+04	1.75777E+04	1.06531E+05
1.13172E-05	1.70355E+04	9.51698E+04	1.96915E+04	1.73413E+04	1.05099E+05
1.14940E-05	1.69115E+04	9.17491E+04	1.97718E+04	1.71418E+04	1.03890E+05
1.16708E-05	1.67922E+04	8.82071E+04	1.98444E+04	1.69807E+04	1.02913E+05
1.18476E-05	1.66776E+04	8.45630E+04	1.98918E+04	1.68579E+04	1.02169E+05
1.20243E-05	1.65679E+04	8.08474E+04	1.99036E+04	1.67720E+04	1.01649E+05
1.22011E-05	1.64632E+04	7.70955E+04	1.98777E+04	1.67205E+04	1.01336E+05
1.23779E-05	1.63634E+04	7.33412E+04	1.98194E+04	1.66995E+04	1.01209E+05
1.25547E-05	1.62687E+04	6.96105E+04	1.97401E+04	1.67044E+04	1.01239E+05
1.27314E-05	1.61788E+04	6.59188E+04	1.96530E+04	1.67297E+04	1.01392E+05
1.29082E-05	1.60938E+04	6.22698E+04	1.95734E+04	1.67697E+04	1.01634E+05
1.30850E-05	1.60136E+04	5.86574E+04	1.95095E+04	1.68182E+04	1.01928E+05
1.32618E-05	1.59382E+04	5.50698E+04	1.94654E+04	1.68691E+04	1.02237E+05
1.34385E-05	1.58676E+04	5.14953E+04	1.94378E+04	1.69165E+04	1.02524E+05
1.36153E-05	1.58016E+04	4.79282E+04	1.94153E+04	1.69549E+04	1.02757E+05
1.37921E-05	1.57404E+04	4.43739E+04	1.93831E+04	1.69795E+04	1.02906E+05
1.39689E-05	1.56839E+04	4.08529E+04	1.93202E+04	1.69859E+04	1.02945E+05
1.41457E-05	1.56320E+04	3.74013E+04	1.92074E+04	1.69707E+04	1.02853E+05
1.43224E-05	1.55847E+04	3.40699E+04	1.90282E+04	1.69312E+04	1.02613E+05
1.44992E-05	1.55416E+04	3.09201E+04	1.87716E+04	1.68657E+04	1.02216E+05
1.46760E-05	1.55025E+04	2.80181E+04	1.84344E+04	1.67732E+04	1.01656E+05
1.48528E-05	1.54671E+04	2.54283E+04	1.80217E+04	1.66536E+04	1.00931E+05
1.51028E-05	1.54224E+04	2.24051E+04	1.73349E+04	1.64398E+04	9.96353E+04
1.53528E-05	1.53826E+04	2.02239E+04	1.65777E+04	1.61777E+04	9.80464E+04
1.56028E-05	1.53460E+04	1.89191E+04	1.58138E+04	1.58738E+04	9.62047E+04
1.58528E-05	1.53111E+04	1.84514E+04	1.51012E+04	1.55366E+04	9.41610E+04
1.61028E-05	1.52763E+04	1.87232E+04	1.44800E+04	1.51754E+04	9.19724E+04
1.63528E-05	1.52405E+04	1.96034E+04	1.39675E+04	1.48002E+04	8.96980E+04
1.66028E-05	1.52025E+04	2.09550E+04	1.35601E+04	1.44202E+04	8.73950E+04
1.68528E-05	1.51617E+04	2.26539E+04	1.32422E+04	1.40439E+04	8.51143E+04
1.71028E-05	1.51174E+04	2.45981E+04	1.29959E+04	1.36762E+04	8.28979E+04
1.73528E-05	1.50693E+04	2.67038E+04	1.28106E+04	1.33281E+04	8.07763E+04
1.76028E-05	1.50172E+04	2.88936E+04	1.26875E+04	1.29967E+04	7.87678E+04
1.78528E-05	1.49609E+04	3.10837E+04	1.26380E+04	1.26850E+04	7.68788E+04
1.81028E-05	1.49006E+04	3.31759E+04	1.26773E+04	1.23923E+04	7.51050E+04
1.83528E-05	1.48366E+04	3.50581E+04	1.28155E+04	1.21166E+04	7.34339E+04
1.86028E-05	1.47693E+04	3.66160E+04	1.30501E+04	1.18546E+04	7.18475E+04
1.88528E-05	1.46995E+04	3.77504E+04	1.33612E+04	1.16037E+04	7.03254E+04
1.91028E-05	1.46280E+04	3.83972E+04	1.37135E+04	1.13599E+04	6.88477E+04
1.93528E-05	1.45558E+04	3.85419E+04	1.40616E+04	1.11206E+04	6.73975E+04
1.96028E-05	1.44838E+04	3.82244E+04	1.43597E+04	1.08839E+04	6.59626E+04
1.98528E-05	1.44127E+04	3.75336E+04	1.45710E+04	1.06488E+04	6.45380E+04
2.01028E-05	1.43431E+04	3.65913E+04	1.46743E+04	1.04156E+04	6.31246E+04
2.03528E-05	1.42755E+04	3.55320E+04	1.46664E+04	1.01856E+04	6.17311E+04
2.06028E-05	1.42099E+04	3.44822E+04	1.45599E+04	9.96148E+03	6.03726E+04
2.08528E-05	1.41461E+04	3.35450E+04	1.43771E+04	9.74631E+03	5.90697E+04
.11028E-05	1.40839E+04	3.27942E+04	1.41623E+04	9.54476E+03	5.78470E+04

2.13528E-05	1.40229E+04	3.22748E+04	1.38779E+04	9.36059E+03	5.67309E+04
2.16028E-05	1.39527E+04	3.20108E+04	1.35357E+04	9.19830E+03	5.57473E+04
2.18528E-05	1.39027E+04	3.20143E+04	1.33021E+04	9.06171E+03	5.49194E+04
2.21028E-05	1.38424E+04	3.22327E+04	1.29988E+04	8.95377E+03	5.42653E+04
2.23528E-05	1.37814E+04	3.28509E+04	1.26876E+04	8.87627E+03	5.37956E+04
2.25295E-05	1.37375E+04	3.34150E+04	1.24659E+04	8.84010E+03	5.35764E+04
2.27063E-05	1.36927E+04	3.41151E+04	1.22473E+04	8.81901E+03	5.34485E+04
2.28831E-05	1.36469E+04	3.49471E+04	1.20375E+04	8.81224E+03	5.34075E+04
2.30599E-05	1.36000E+04	3.58955E+04	1.18434E+04	8.81867E+03	5.34465E+04
2.32366E-05	1.35517E+04	3.69432E+04	1.16722E+04	8.83631E+03	5.35564E+04
2.34134E-05	1.35019E+04	3.80561E+04	1.15306E+04	8.86487E+03	5.37265E+04
2.35902E-05	1.34507E+04	3.92358E+04	1.14239E+04	8.90078E+03	5.39442E+04
2.37670E-05	1.33979E+04	4.04202E+04	1.13554E+04	8.94232E+03	5.41959E+04
2.39437E-05	1.33435E+04	4.15864E+04	1.13255E+04	8.98711E+03	5.44673E+04
2.41205E-05	1.32876E+04	4.27026E+04	1.13319E+04	9.03275E+03	5.47440E+04
2.43705E-05	1.32061E+04	4.41417E+04	1.13925E+04	9.09416E+03	5.51161E+04
2.46205E-05	1.31222E+04	4.53643E+04	1.14950E+04	9.14624E+03	5.54318E+04
2.48705E-05	1.30361E+04	4.63370E+04	1.16154E+04	9.18347E+03	5.56574E+04
2.51205E-05	1.29485E+04	4.70544E+04	1.17318E+04	9.20147E+03	5.57665E+04
2.53705E-05	1.28598E+04	4.75343E+04	1.18276E+04	9.19719E+03	5.57406E+04
2.56205E-05	1.27703E+04	4.78093E+04	1.18941E+04	9.16913E+03	5.55705E+04
2.58705E-05	1.26806E+04	4.79174E+04	1.19298E+04	9.11736E+03	5.52567E+04
2.61205E-05	1.25907E+04	4.78951E+04	1.19383E+04	9.04354E+03	5.48093E+04
2.63705E-05	1.25010E+04	4.77730E+04	1.19249E+04	8.95075E+03	5.42470E+04
2.66205E-05	1.24115E+04	4.75755E+04	1.18941E+04	8.84332E+03	5.35959E+04
2.68705E-05	1.23225E+04	4.73234E+04	1.18473E+04	8.72649E+03	5.28878E+04
2.71205E-05	1.22341E+04	4.70367E+04	1.17833E+04	8.60606E+03	5.21579E+04
2.73705E-05	1.21461E+04	4.67374E+04	1.17011E+04	8.48799E+03	5.14423E+04
2.76205E-05	1.20587E+04	4.64492E+04	1.15984E+04	8.37799E+03	5.07757E+04
2.78705E-05	1.19719E+04	4.61948E+04	1.14776E+04	8.28117E+03	5.01889E+04
2.81205E-05	1.18854E+04	4.59917E+04	1.13452E+04	8.20167E+03	4.97071E+04
2.83705E-05	1.17993E+04	4.58473E+04	1.12113E+04	8.14243E+03	4.93481E+04
2.86205E-05	1.17134E+04	4.57556E+04	1.10880E+04	8.10506E+03	4.91216E+04
2.88705E-05	1.16277E+04	4.56970E+04	1.09867E+04	8.08976E+03	4.90289E+04
2.91205E-05	1.15420E+04	4.56405E+04	1.09154E+04	8.09537E+03	4.90629E+04
2.93705E-05	1.14565E+04	4.55497E+04	1.08766E+04	8.11952E+03	4.92092E+04
2.96205E-05	1.13712E+04	4.53890E+04	1.08673E+04	8.15878E+03	4.94472E+04
2.98705E-05	1.12863E+04	4.51306E+04	1.08792E+04	8.20898E+03	4.97514E+04
3.01205E-05	1.12020E+04	4.47583E+04	1.09016E+04	8.26543E+03	5.00935E+04
3.03705E-05	1.11185E+04	4.42693E+04	1.09230E+04	8.32327E+03	5.04440E+04
3.06205E-05	1.10360E+04	4.36736E+04	1.09344E+04	8.37775E+03	5.07742E+04
3.08705E-05	1.09547E+04	4.29896E+04	1.09294E+04	8.42456E+03	5.10579E+04
3.11205E-05	1.08748E+04	4.22411E+04	1.09057E+04	8.46003E+03	5.12729E+04
3.12973E-05	1.08192E+04	4.16861E+04	1.08775E+04	8.47670E+03	5.13739E+04
3.14741E-05	1.07642E+04	4.11195E+04	1.08405E+04	8.48557E+03	5.14277E+04
3.16508E-05	1.07101E+04	4.05490E+04	1.07951E+04	8.48623E+03	5.14317E+04
3.18276E-05	1.06567E+04	3.99818E+04	1.07420E+04	8.47850E+03	5.13849E+04
3.20044E-05	1.06040E+04	3.94247E+04	1.06813E+04	8.46249E+03	5.12878E+04
3.21812E-05	1.05521E+04	3.88845E+04	1.06130E+04	8.43854E+03	5.11427E+04
3.23580E-05	1.05009E+04	3.83677E+04	1.05369E+04	8.40721E+03	5.09528E+04
3.25347E-05	1.04503E+04	3.78811E+04	1.04529E+04	8.36926E+03	5.07228E+04
3.27115E-05	1.04004E+04	3.74313E+04	1.03610E+04	8.32560E+03	5.04582E+04
3.28883E-05	1.03510E+04	3.70246E+04	1.02617E+04	8.27727E+03	5.01653E+04
3.31383E-05	1.02821E+04	3.65337E+04	1.01106E+04	8.20308E+03	4.97157E+04
3.34918E-05	1.01859E+04	3.60257E+04	9.88635E+03	8.09214E+03	4.90433E+04
3.38454E-05	1.00906E+04	3.57365E+04	9.66945E+03	7.98172E+03	4.83741E+04
3.40954E-05	1.00239E+04	3.56458E+04	9.53406E+03	7.90781E+03	4.79261E+04
3.43454E-05	9.95704E+03	3.56227E+04	9.42202E+03	7.83944E+03	4.75116E+04
3.45954E-05	9.89022E+03	3.56363E+04	9.33779E+03	7.77771E+03	4.71377E+04
3.48454E-05	9.82337E+03	3.56531E+04	9.28222E+03	7.72305E+03	4.68064E+04
3.50954E-05	9.75650E+03	3.56409E+04	9.25303E+03	7.67519E+03	4.65163E+04
3.53454E-05	9.68971E+03	3.55727E+04	9.24522E+03	7.63329E+03	4.62624E+04
3.55222E-05	9.64260E+03	3.54794E+04	9.24918E+03	7.60656E+03	4.61003E+04
3.56989E-05	9.59563E+03	3.53432E+04	9.25833E+03	7.58155E+03	4.59488E+04
3.58757E-05	9.54868E+03	3.51617E+04	9.27048E+03	7.55765E+03	4.58039E+04
3.60525E-05	9.50239E+03	3.49337E+04	9.28370E+03	7.53417E+03	4.56617E+04
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9.50343E-05	2.08495E+03	7.18315E+03	2.02037E+03	1.59821E+03	9.68611E+03
9.52843E-05	2.07152E+03	7.13287E+03	2.00780E+03	1.58731E+03	9.62367E+03
9.55343E-05	2.05819E+03	7.08263E+03	1.99509E+03	1.57755E+03	9.56092E+03
9.57843E-05	2.04496E+03	7.03277E+03	1.98210E+03	1.56716E+03	9.49793E+03
9.60343E-05	2.03181E+03	6.98353E+03	1.96399E+03	1.55676E+03	9.43489E+03
9.62843E-05	2.01876E+03	6.93534E+03	1.95574E+03	1.54636E+03	9.37189E+03
9.64611E-05	2.00959E+03	6.90192E+03	1.94633E+03	1.53903E+03	9.32748E+03
9.66379E-05	2.00046E+03	6.86914E+03	1.93689E+03	1.53174E+03	9.28328E+03
9.68146E-05	1.99137E+03	6.83703E+03	1.92745E+03	1.52449E+03	9.23935E+03
9.69914E-05	1.98232E+03	6.80561E+03	1.91804E+03	1.51729E+03	9.19571E+03
9.71682E-05	1.97332E+03	6.77487E+03	1.90867E+03	1.51016E+03	9.15247E+03
9.73450E-05	1.96435E+03	6.74479E+03	1.89938E+03	1.50309E+03	9.10965E+03
9.75217E-05	1.95543E+03	6.71532E+03	1.89019E+03	1.49610E+03	9.06725E+03
9.76985E-05	1.94654E+03	6.68640E+03	1.88110E+03	1.48918E+03	9.02535E+03
9.79485E-05	1.93404E+03	6.64631E+03	1.86847E+03	1.47955E+03	8.96695E+03
9.81985E-05	1.92161E+03	6.60695E+03	1.85613E+03	1.47007E+03	8.90953E+03
9.84485E-05	1.90926E+03	6.56303E+03	1.84408E+03	1.46076E+03	8.85308E+03
9.86985E-05	1.89697E+03	6.52930E+03	1.83233E+03	1.45158E+03	8.79748E+03
9.89485E-05	1.88477E+03	6.49051E+03	1.82086E+03	1.44253E+03	8.74264E+03
9.91985E-05	1.87263E+03	6.45143E+03	1.80964E+03	1.43358E+03	8.68839E+03
9.94485E-05	1.86057E+03	6.41190E+03	1.79862E+03	1.42471E+03	8.63461E+03
9.96985E-05	1.84858E+03	6.37181E+03	1.78774E+03	1.41589E+03	8.58113E+03
9.99485E-05	1.83667E+03	6.33110E+03	1.77697E+03	1.40709E+03	8.52780E+03
1.00199E-04	1.82483E+03	6.28981E+03	1.76624E+03	1.39829E+03	8.47448E+03
1.00449E-04	1.81307E+03	6.24799E+03	1.75550E+03	1.38948E+03	8.42110E+03
1.00699E-04	1.80140E+03	6.20580E+03	1.74471E+03	1.38065E+03	8.36756E+03
1.00949E-04	1.78980E+03	6.16339E+03	1.73385E+03	1.37179E+03	8.31367E+03
1.01199E-04	1.77828E+03	6.12098E+03	1.72289E+03	1.36290E+03	8.26003E+03
1.01449E-04	1.76684E+03	6.07876E+03	1.71181E+03	1.35401E+03	8.20611E+03
1.01699E-04	1.75548E+03	6.03696E+03	1.70064E+03	1.34510E+03	8.15213E+03
1.01949E-04	1.74419E+03	5.99575E+03	1.68939E+03	1.33621E+03	8.09822E+03
1.02199E-04	1.73299E+03	5.95530E+03	1.67808E+03	1.32734E+03	8.04448E+03
1.02449E-04	1.72185E+03	5.91571E+03	1.66676E+03	1.31852E+03	7.99106E+03
1.02699E-04	1.71080E+03	5.87707E+03	1.65547E+03	1.30978E+03	7.93809E+03
1.02949E-04	1.69981E+03	5.83937E+03	1.64424E+03	1.30113E+03	7.88564E+03
1.03199E-04	1.68889E+03	5.80258E+03	1.63313E+03	1.29256E+03	7.83382E+03
1.03449E-04	1.67804E+03	5.76661E+03	1.62215E+03	1.28414E+03	7.78268E+03
1.03625E-04	1.67041E+03	5.74160E+03	1.61452E+03	1.27825E+03	7.74698E+03
1.03802E-04	1.66282E+03	5.71688E+03	1.60697E+03	1.27243E+03	7.71168E+03
1.03979E-04	1.65525E+03	5.69240E+03	1.59953E+03	1.26666E+03	7.67672E+03
1.04156E-04	1.64772E+03	5.66809E+03	1.59219E+03	1.26095E+03	7.64211E+03
1.04332E-04	1.64022E+03	5.64388E+03	1.58495E+03	1.25529E+03	7.60784E+03
1.04509E-04	1.63275E+03	5.61973E+03	1.57782E+03	1.24969E+03	7.57389E+03
1.04686E-04	1.62531E+03	5.59556E+03	1.57079E+03	1.24413E+03	7.54021E+03
1.04863E-04	1.61791E+03	5.57135E+03	1.56385E+03	1.23862E+03	7.50678E+03
1.05040E-04	1.61053E+03	5.54703E+03	1.55698E+03	1.23314E+03	7.47357E+03
1.05216E-04	1.60319E+03	5.52257E+03	1.55019E+03	1.22769E+03	7.44053E+03
1.05466E-04	1.59287E+03	5.48772E+03	1.54067E+03	1.22002E+03	7.39404E+03
1.05820E-04	1.57838E+03	5.43786E+03	1.52734E+03	1.20921E+03	7.32856E+03
1.06173E-04	1.56402E+03	5.38737E+03	1.51407E+03	1.19844E+03	7.26325E+03
1.06527E-04	1.54980E+03	5.33647E+03	1.50077E+03	1.18765E+03	7.19788E+03
1.06704E-04	1.54274E+03	5.31096E+03	1.49410E+03	1.18225E+03	7.16517E+03
1.06881E-04	1.53572E+03	5.28548E+03	1.48739E+03	1.17685E+03	7.13245E+03
1.07057E-04	1.52872E+03	5.26007E+03	1.48066E+03	1.17145E+03	7.09972E+03
1.07234E-04	1.52177E+03	5.23478E+03	1.47390E+03	1.16605E+03	7.06698E+03
1.07411E-04	1.51484E+03	5.20967E+03	1.46712E+03	1.16066E+03	7.03428E+03
1.07588E-04	1.50795E+03	5.18476E+03	1.46031E+03	1.15527E+03	7.00163E+03
1.07764E-04	1.50109E+03	5.16012E+03	1.45349E+03	1.14989E+03	6.96905E+03
1.07941E-04	1.49426E+03	5.13576E+03	1.44666E+03	1.14453E+03	6.93657E+03
1.08118E-04	1.48747E+03	5.11172E+03	1.43983E+03	1.13919E+03	6.90420E+03
1.08295E-04	1.48070E+03	5.08801E+03	1.43301E+03	1.13388E+03	6.87200E+03
1.08545E-04	1.47119E+03	5.05506E+03	1.42341E+03	1.12642E+03	6.82676E+03
1.08898E-04	1.45784E+03	5.00961E+03	1.40995E+03	1.11599E+03	6.76356E+03
1.09252E-04	1.44462E+03	4.96538E+03	1.39671E+03	1.10572E+03	6.70136E+03
1.09502E-04	1.43533E+03	4.93471E+03	1.38759E+03	1.09858E+03	6.65903E+03
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

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⋮	⋮	⋮	⋮	⋮	⋮
2.72946E-04	2.12568E+01	7.25841E+01	2.05793E+01	1.63069E+01	9.88295E+01
2.73122E-04	2.11608E+01	7.22468E+01	2.04855E+01	1.62372E+01	9.84071E+01
2.73372E-04	2.10257E+01	7.17751E+01	2.03520E+01	1.61380E+01	9.78097E+01
2.73726E-04	2.08362E+01	7.11198E+01	2.01635E+01	1.59992E+01	9.69549E+01
2.74079E-04	2.06485E+01	7.04790E+01	1.99763E+01	1.58540E+01	9.60848E+01
2.74329E-04	2.05167E+01	7.00337E+01	1.98467E+01	1.57472E+01	9.54376E+01
2.74579E-04	2.03858E+01	6.95935E+01	1.97184E+01	1.56425E+01	9.48029E+01
2.74829E-04	2.02557E+01	6.91574E+01	1.95920E+01	1.55398E+01	9.41806E+01
2.75079E-04	2.01264E+01	6.87240E+01	1.94674E+01	1.54412E+01	9.35832E+01
2.75329E-04	1.99979E+01	6.82925E+01	1.93438E+01	1.53427E+01	9.29858E+01
2.75579E-04	1.98702E+01	6.78632E+01	1.92206E+01	1.52400E+01	9.23635E+01
2.75756E-04	1.97804E+01	6.75612E+01	1.91333E+01	1.51674E+01	9.19235E+01
2.75933E-04	1.96910E+01	6.72608E+01	1.90465E+01	1.51006E+01	9.15187E+01
2.76110E-04	1.96020E+01	6.69621E+01	1.89597E+01	1.50309E+01	9.10962E+01
2.76286E-04	1.95134E+01	6.66653E+01	1.88731E+01	1.49612E+01	9.06738E+01
2.76463E-04	1.94252E+01	6.63706E+01	1.87867E+01	1.48915E+01	9.02514E+01
2.76640E-04	1.93374E+01	6.60780E+01	1.87008E+01	1.48218E+01	8.98290E+01
2.76817E-04	1.92499E+01	6.57874E+01	1.86156E+01	1.47521E+01	8.94065E+01
2.76994E-04	1.91629E+01	6.54984E+01	1.85310E+01	1.46824E+01	8.89841E+01
2.77170E-04	1.90762E+01	6.52107E+01	1.84472E+01	1.46127E+01	8.85617E+01
2.77347E-04	1.89899E+01	6.49240E+01	1.83643E+01	1.45430E+01	8.81393E+01
2.77597E-04	1.88685E+01	6.45194E+01	1.82484E+01	1.44567E+01	8.75165E+01
2.77951E-04	1.86982E+01	6.39476E+01	1.80870E+01	1.43348E+01	8.68773E+01
2.78304E-04	1.85293E+01	6.33744E+01	1.79279E+01	1.42128E+01	8.61381E+01
2.78658E-04	1.83620E+01	6.27991E+01	1.77706E+01	1.40821E+01	8.53460E+01
2.78835E-04	1.82789E+01	6.25107E+01	1.76920E+01	1.40182E+01	8.49588E+01
2.79011E-04	1.81962E+01	6.22222E+01	1.76140E+01	1.39601E+01	8.46068E+01
2.79188E-04	1.81139E+01	6.19334E+01	1.75361E+01	1.38991E+01	8.42372E+01
2.79365E-04	1.80319E+01	6.16447E+01	1.74582E+01	1.38352E+01	8.38499E+01
2.79542E-04	1.79504E+01	6.13564E+01	1.73802E+01	1.37743E+01	8.34803E+01
2.79718E-04	1.78692E+01	6.10689E+01	1.73022E+01	1.37133E+01	8.31107E+01
2.79895E-04	1.77884E+01	6.07825E+01	1.72241E+01	1.36523E+01	8.27411E+01
2.80072E-04	1.77080E+01	6.04977E+01	1.71459E+01	1.35913E+01	8.23714E+01
2.80249E-04	1.76279E+01	5.02147E+01	1.70676E+01	1.35303E+01	8.20018E+01
2.80426E-04	1.75483E+01	5.99340E+01	1.69892E+01	1.34693E+01	8.16322E+01
2.80676E-04	1.74362E+01	5.95413E+01	1.68784E+01	1.33831E+01	8.11095E+01
2.81029E-04	1.72790E+01	5.89959E+01	1.67222E+01	1.32611E+01	8.03702E+01
2.81383E-04	1.71233E+01	5.84622E+01	1.65676E+01	1.31391E+01	7.96310E+01
2.81633E-04	1.70140E+01	5.80913E+01	1.64596E+01	1.30508E+01	7.90958E+01
2.81883E-04	1.69054E+01	5.77248E+01	1.63520E+01	1.29666E+01	7.85856E+01
2.82133E-04	1.67974E+01	5.73618E+01	1.62479E+01	1.28845E+01	7.80677E+01
2.82383E-04	1.66902E+01	5.70013E+01	1.61444E+01	1.28044E+01	7.76023E+01
2.82633E-04	1.65836E+01	5.66427E+01	1.60419E+01	1.27181E+01	7.70796E+01
2.82883E-04	1.64777E+01	5.62855E+01	1.59400E+01	1.26319E+01	7.65569E+01
2.83059E-04	1.64033E+01	5.60340E+01	1.58680E+01	1.25724E+01	7.61961E+01
2.83236E-04	1.63291E+01	5.57835E+01	1.57964E+01	1.25186E+01	7.58705E+01
2.83413E-04	1.62553E+01	5.55340E+01	1.57252E+01	1.24634E+01	7.55360E+01
2.83590E-04	1.61818E+01	5.52855E+01	1.56540E+01	1.24083E+01	7.52016E+01
2.83767E-04	1.61087E+01	5.50383E+01	1.55831E+01	1.23531E+01	7.48672E+01
2.83943E-04	1.60358E+01	5.47923E+01	1.55125E+01	1.23008E+01	7.45504E+01
2.84120E-04	1.59633E+01	5.45474E+01	1.54423E+01	1.22485E+01	7.42336E+01
2.84297E-04	1.58912E+01	5.43035E+01	1.53726E+01	1.21963E+01	7.39167E+01
2.84474E-04	1.58193E+01	5.40605E+01	1.53034E+01	1.21440E+01	7.35999E+01
2.84650E-04	1.57478E+01	5.38183E+01	1.52348E+01	1.20917E+01	7.32831E+01
2.84900E-04	1.56472E+01	5.34765E+01	1.51386E+01	1.20178E+01	7.28351E+01
2.85254E-04	1.55060E+01	5.29941E+01	1.50044E+01	1.19132E+01	7.22014E+01
2.85608E-04	1.53661E+01	5.25116E+01	1.48721E+01	1.18087E+01	7.15676E+01
2.85961E-04	1.52274E+01	5.20281E+01	1.47415E+01	1.16983E+01	7.08969E+01
2.86138E-04	1.51586E+01	5.17859E+01	1.46766E+01	1.16431E+01	7.05645E+01
2.86315E-04	1.50901E+01	5.15432E+01	1.46123E+01	1.15938E+01	7.02653E+01
2.86491E-04	1.50219E+01	5.13000E+01	1.45483E+01	1.15415E+01	6.99485E+01
2.86668E-04	1.49540E+01	5.10566E+01	1.44844E+01	1.14892E+01	6.95317E+01
2.86845E-04	1.48865E+01	5.08129E+01	1.44206E+01	1.14370E+01	6.93149E+01
2.87022E-04	1.48192E+01	5.05693E+01	1.43569E+01	1.13847E+01	6.89980E+01
2.87199E-04	1.47523E+01	5.03260E+01	1.42931E+01	1.13324E+01	6.86812E+01
2.87376E-04	1.46858E+01	5.00833E+01	1.42291E+01	1.12801E+01	6.83644E+01
⋮	⋮	⋮	⋮	⋮	⋮

3.62032E-04	2.23520E+00	7.42474E+00	2.15210E+00	1.71905E+00	1.04185E+01
3.62209E-04	2.22537E+00	7.40025E+00	2.14569E+00	1.70888E+00	1.03569E+01
3.62385E-04	2.21557E+00	7.37320E+00	2.13942E+00	1.70307E+00	1.03217E+01
3.62562E-04	2.20581E+00	7.34365E+00	2.13300E+00	1.70017E+00	1.03041E+01
3.62739E-04	2.19610E+00	7.31195E+00	2.12608E+00	1.69146E+00	1.02513E+01
3.62916E-04	2.18642E+00	7.27861E+00	2.11857E+00	1.68275E+00	1.01985E+01
3.63092E-04	2.17679E+00	7.24416E+00	2.11049E+00	1.67403E+00	1.01457E+01
3.63269E-04	2.16721E+00	7.20911E+00	2.10188E+00	1.66532E+00	1.00929E+01
3.63446E-04	2.15767E+00	7.17392E+00	2.09279E+00	1.65661E+00	1.00401E+01
3.63623E-04	2.14818E+00	7.13895E+00	2.08333E+00	1.64790E+00	9.98725E+00
3.63873E-04	2.13484E+00	7.09047E+00	2.06959E+00	1.63557E+00	9.91257E+00
3.64123E-04	2.12156E+00	7.04344E+00	2.05568E+00	1.62325E+00	9.83790E+00
3.64373E-04	2.10842E+00	6.99803E+00	2.04181E+00	1.61093E+00	9.76322E+00
3.64623E-04	2.09533E+00	6.95420E+00	2.02805E+00	1.59851E+00	9.68855E+00
3.64976E-04	2.07697E+00	6.89486E+00	2.00865E+00	1.58119E+00	9.58294E+00
3.65330E-04	2.05875E+00	6.83807E+00	1.98890E+00	1.56957E+00	9.51254E+00
3.65683E-04	2.04068E+00	6.78742E+00	1.96847E+00	1.56086E+00	9.45973E+00
3.65933E-04	2.02799E+00	6.75413E+00	1.95421E+00	1.55470E+00	9.42240E+00
3.66110E-04	2.01904E+00	6.73201E+00	1.94434E+00	1.54889E+00	9.38720E+00
3.66287E-04	2.01013E+00	6.71103E+00	1.93428E+00	1.54017E+00	9.33439E+00
3.66464E-04	2.00124E+00	6.69115E+00	1.92447E+00	1.53437E+00	9.29919E+00
3.66641E-04	1.99238E+00	6.67210E+00	1.91509E+00	1.53146E+00	9.28159E+00
3.66817E-04	1.98355E+00	6.65358E+00	1.90606E+00	1.52565E+00	9.24639E+00

Appendix B Parameter Estimation Program

Operating Instructions for ZXMIN and MEXP,
Flow Chart, and Estimation Program Source Listing

Operating Instructions for ZXMIN (Ref 5)

```
                    SUBROUTINE ZXMIN  (FUNCT,N,NSIG,MAXFN,IOPT,X,H,G,F,W,IER)
C
C   FUNCTION                      - A QUASI-NEWTON ALGORITHM FOR FINDING THE
C                                     MINIMUM OF A FUNCTION OF N VARIABLES.
C   USAGE                         - CALL ZXMIN (FUNCT,N, NSIG, MAXFN,IOPT,X,H,G,F,
C                                     W,IER)
C   PARAMETERS    FUNCT           - A USER SUPPLIED SUBROUTINE WHICH CALCULATES
C                                     THE FUNCTION F FOR GIVEN PARAMETER VALUES
C                                     X (1) ,X(2),,,,,X(N) .
C                                     THE CALLING SEQUENCE HAS THE FOLLOWING FORM
C                                     CALL FUNCT (N,X,F)
C                                     WHERE X IS A VECTOR OF LENGTH N.
C                                     FUNCT MUST APPEAR IN AN EXTERNAL STATEMENT
C                                     IN THE CALLING PROGRAM.  FUNCT MUST NOT
C                                     ALTER THE VALUES OF X(I),I=1,,,,,N OR N.
C                 N               - THE NUMBER OF PARAMETERS (I.E.. THE LENGTH
C                                     OF X) (INPUT)
C                 NSIG            - CONVERGENCE CRITERION.  (INPUT). THE NUMBER
C                                     OF DIGITS OF ACCURACY REQUIRED IN THE
C                                     PARAMETER ESTIMATES.
C                                     THIS CONVERGENCE CONDITION IS SATISIFIED IF
C                                     ON TWO SUCCESSIVE ITERATIONS. THE PARAMETER
C                                     ESTIMATES (I,E,,X(I), I=1,,,,,N) AGREE.
C                                     COMPONENT BY COMPONENT, TO NSIG DIGITS.
C                 MAXFN           - MAXIMUM NUMBER OF FUNCTION EVALUATIONS (I.E..
C                                     CALLS TO SUBROUTINE FUNCT) ALLOWED, (INPUT)
C                 IOPT            - INPUT OPTIONS SELECTOR.
C                                   IOPT = 0 CAUSES ZXMIN TO INITIALIZE THE
C                                     HESSIAN MATRIX H TO THE IDENTITY MATRIX.
C                                   IOPT = 1 INDICATES THAT H HAS BEEN INITIALIZED
C                                     BY THE USER TO A POSITIVE DEFINITE MATRIX.
C                 X               - VECTOR OF LENGTH N CONTAINING PARAMETER
C                                     VALUES.
C                                   ON INPUT, X MUST CONTAIN THE INITIAL
C                                     PARAMETER ESTIMATES.
C                                   ON OUTPUT, X CONTAINS THE FINAL PARAMETER
C                                     ESTIMATES AS DETERMINED BY ZXMIN.
C                 H               - VECTOR OF LENGTH N* (N+1)/2 CONTAINING AN
C                                     ESTIMATE OF THE HESSIAN MATRIX
C                                     D**2F/(DX(I)DX(J)), I,J=1,...,N.
C                                     H IS STORED IN SYMMETRIC STORAGE MODE.
C                                   ON INPUT, IF IOPT = 0, ZXMIN INITIALIZES H
C                                     TO THE IDENTITY MATRIX.  AN INITIAL SETTING
C                                     OF H BY THE USER IS INDICATED BY IOPT=1.
C                                     H MUST BE POSITIVE DEFINITE.  IF IT IS NOT,
C                                     A TERMINAL ERROR OCCURS.
C                                   ON OUTPUT,H CONTAINS AN ESTIMATE OF THE
C                                     HESSIAN AT THE FINAL PARAMETER ESTIMATES
C                                     (I.E., AT X (1),X (2),...X(N)
C                 G               - A VECTOR OF LENGTH N CONTAINING AN ESTIMATE
C                                     OF THE GRADIENT DF/DX(I), I=1,...,N AT THE
C                                     FINAL PARAMETER ESTIMATES.  (OUTPUT)
C                 F               - A SCALAR CONTAINING THE VALUE OF THE FUNCTION
C                                     AT THE FINAL PARAMETER ESTIMATES.  (OUTPUT)
C                 W               - A VECTOR OF LENGTH 3*N USED AS WORKING SPACE.
C                                     ON OUTPUT, WORK (I), CONTAINS FOR I=1, THE NORM
C                                     OF THE GRADIENT (I.E.,SQRT(G(1)**2+G(2)**2+...+G(N)**2)
```

```
C                                              I=2, THE NUMBER OF FUNCTION EVALTIONS
C                                              PERFORMED.
C                                              I=3, AN ESTIMATE OF THE NUMBER OF
C                                                 SIGNIFICANT DIGITS IN THE FINAL
C                                                 PARAMETER ESTIMATES.
C                   IER                     - ERROR PARAMETER.
C                                            IER =0 IMPLIES THAT CONVERGENCE WAS
C                                              ACHIEVED AND NO ERRORS OCCURRED.
C                                            TERMINAL ERROR
C                                              IER = 129 IMPLIES THAT THE INITIAL HESSIAN
C                                                 MATRIX IS NOT POSITIVE DEFINITE.  THIS
C                                                 CAN OCCUR ONLY FOR IOPT = 1.
C                                              IER - 130 IMPLIES THAT THE INTERATION WAS
C                                                 TERMINATED DUE TO ROUNDING ERRORS
C                                                 BECOMING DOMINANT.  THE PARAMETER
C                                                 ESTIMATES HAVE NOT BEEN DETERMINED TO
C                                                 NSIG DIGITS.
C                                              IER - 131 IMPLIES THAT THE INTERATION WAS
C                                                 TERMINATED BECAUSE MAXFN WAS EXCEEDED.
C    PRECISION                              - SINGLE
C    REQD. IMSL ROUTINES                    - UERTST,ZXMJN
C    LANGUAGE                               - FORTRAN
C-----------------------------------------------------------------------------
```

CALL ZXMIN (FUNCT,N,NSIG,MAXFN,IOPT,X,H,G,F,W,IER)

Purpose

To find the minimum of a function f(x) of N variables $\underline{x} = (x_1, x_2, \ldots, x_N)$. It is assumed that the gradient vector $g(\underline{x}) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \cdots \frac{\partial f}{\partial x_N}\right)$ and the Hessian matrix $H = \left(\frac{\partial^2 f}{\partial x_i \partial x_j}\right)$ exist, but the user is not required to supply formulas for their evaluation.

Algorithm

ZXMIN is based on the Harwell library routine VA10A. It uses a quasi-Newton method.

See reference: Fletcher, R., "Fortran subroutines for minimization by quasi-Newton methods", Report R7125 AERE, Harwell, England.

Programming Notes

1. Accuracy in evaluating the function f is critical when highly accurate parameter values, x_i, are required (i.e., when NSIG is greater than 3 for single precision). In these cases, it is advisable to evaluate f in precision greater than working precision (e.g., for single precision AXMIN, double precision can be used).

2. In reference to significant digit tests for the x vector, leading zeroes to the right of the decimal point are counted. For example, both 123.45 and 0.00123 have five significant digits. Scaling the x vector may be required if several of the parameters x_j are much less than 1.0 to obtain the same number of significant digits for each of the x_j.

3. The user can place a limit on the amount of computer time used by ZXMIN by setting MAXFN appropriately. In general, MAXFN=500 is satisfactory with NSIG=3.

4. The accuracy of the final Hessian matrix, H, is not always satisfactory. For example when a minimum is located in a very few steps, the estimate may be inaccurate. H is calculated by updating the initial Hessian as steps occur.

<u>Example</u>

Suppose that $f=100(x_2-x_1^2)^2 + (1-x_1)^2$ is to be minimized and parameter estimates x_1 and x_2 are to have 3 digit accuracy.

Input:

```
      EXTERNAL      FUNCT
      DIMENSION     X(2), H(3), G(2), W(6)
      N=2
      NSIG=3
      MAXFN=500
      IOPT=0
      X(1)=-1.2
      X(2)=1.0
        .
        .
        .
      Call ZXMIN(FUNCT,N,NSIG,MAXFN,IOPT,X,H,G,F,W,IER)
```

where subroutine FUNCT has the following code:

```
      SUBROUTINE FUNCT (N,X,F)
      DIMENSION  X(N)
      F=100.* (X(2)-X(1)*X(1))**2+(1.-X(1))**2
      RETURN
      END
```

Output:

IER = 0
X(1) = .9999
X(2) = .9999
10^{-8}
G(1) 10^{-3}
G(2) 10^{-3}
W(1) 10^{-3} (norm of gradient)
W(2) 200 (number of evaluations)
W(3) 3 (estimate of significant digits in $\underline{x}$)

OPERATING INSTRUCTIONS FOR MEXP (REF 6)

INTRODUCTION

This manual is a guide to using various computer programs that have application in linear multivariable systems studies. The programs are described in detail. The accompanying FORTRAN IV listings are available.

The computational algorithms and computer programs were coded by the author over the years 1967 - 1975. They range in scope from simple matrix multiplication routines to complex routines for solving matrix Riccati equations. The coding is highly efficient with considerable attention given to computational speed. Thus, all matrices are treated internally as vector arrays in order to reduce the inherent overhead cost of dual indexing.

The programs have been used successfully for several years. Their worth and efficiency have been amply demonstrated in numerous applications.

Labeled COMMON Blocks

The various subroutines require certain information to be made available through named COMMON blocks. In addition, dummy workspace is to be allocated in the MAIN program. The blocks required are:

1. COMMON/MAIN1/NDIM, NDIM1, COM1

This block is defined in the main program where

NDIM	=	FORTRAN specified dimension of square arrays. This is the number as specified in the DIMENSION declaration. At present, NDIM must be 80 or less.
NDIM1	=	NDIM+1
COM1	=	A dummy workspace (scratch-pad) array. COM1 must be dimensioned in the main program to be of dimension NDIM X NDIM.

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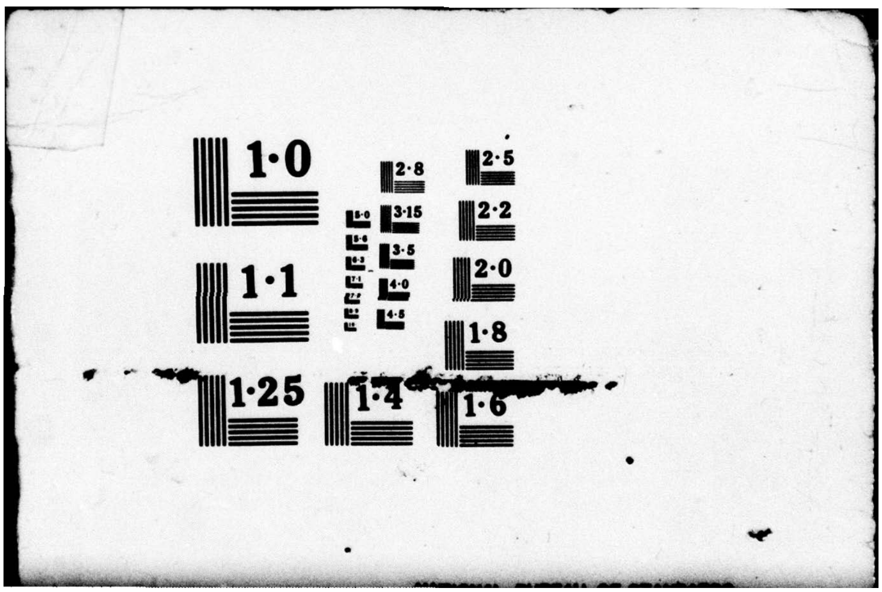
1·0
2·8
2·5
3·15
2·2
3·5
2·0
1·1
4·0
4·5
1·8
1·25
1·4
1·6

2. COMMON/MAIN2/COM2

Defined in the main program. COM2 is an array containing at least NDIM X NDIM locations.

3. COMMON/INOU/KIN, KOUT, KPUNCH

Defined in the main program for input-output control.

KIN = logical unit number input device
KOUT = logical unit number output device
KPUNCH = logical unit number for punch

The various subroutines generally do not output any results through COMMON blocks. The transferring of data and computational results are accomplished via the calling sequences.

Typical first few cards in the main program are:

```
DIMENSION COM1 (20,20), COM2 (20,20),...(additional arrays)
COMMON/MAIN1/NDIM, NDIM1,COM1/MAIN2/COM2/INOU/KIN,KOUT,KPUNCH
NDIM=20
NDIM1=21
KIN=5
KOUT=6
KPUNCH=7
 .
 .
 .
```

The maximum allowable dimension, NDIM = 80.

SUBROUTINE MEXP

Purpose - Compute the NxN matrix exponential

$$EA = e^{AT} \tag{1}$$

using a Chebyshev polynomial approximation.

Calling Sequence - CALL MEXP (N,A,T,EA)

Inputs:

N = size of A matrix (NxN)

A = parameter array

T = scalar

Outputs:

EA = matrix exponential

Other Programs Used: IDENT, XNORM, MMUL, EQUATE, DIAG

Labeled COMMON block: /MAIN1/

Computational Algorithm: The Chebyshev approximation

$$e^{2x} = I_o(2) + 2\sum_{m=1}^{n} I_m(2).T_m(x) \qquad |x| \le 1 \tag{2}$$

$$= \sum_{i=0}^{n} c_i x^i$$

is used to obtain an $n = 12^{th}$ order approximation to e^{2x} where

$I_m(2)$ = modified Bessel function of the first kind

$$= \sum_{k=0}^{\infty} \frac{1}{k!\ (m+k)!} \tag{3}$$

and

$T_m(x)$ = Chebyshev polynomial

$$= \frac{m}{2} \sum_{s=0}^{2s\leq m} \frac{(-1)^s}{m-s} \begin{bmatrix} m-s \\ s \end{bmatrix} (2x)^{m-2s} \tag{4}$$

The error in the approximation (2) is given by

$$\left|e_n(x)\right| \leq 2I_n(2) \cdot \frac{n+1}{n} \qquad |x| \leq 1 \tag{5}$$

Thus, for n = 12, an accuracy of

$$\left|e_{12}(x)\right| \leq 2.3 \times 10^{-9}$$

is achievable, theoretically.

To obtain the matrix e^{AT}, an $M \geq 1$ is found such that

$$\frac{\|A\|\,T}{2^M} \leq 1 \tag{6}$$

Eq. (2) is then used to obtain

$$X = e^{2 \cdot \frac{AT}{2^M}} = e^{AT/2^{M-1}} = \sum_{i=0}^{n} c_i \left[\frac{AT}{2^M}\right]^i \tag{7}$$

Finally e^{AT} is found by $E\Lambda = X^{2^{M-1}}$. The norm of the resulting error matrix (theoretically) satisfies $\|E\| \leq 2.3 \times 10^{-9}$.

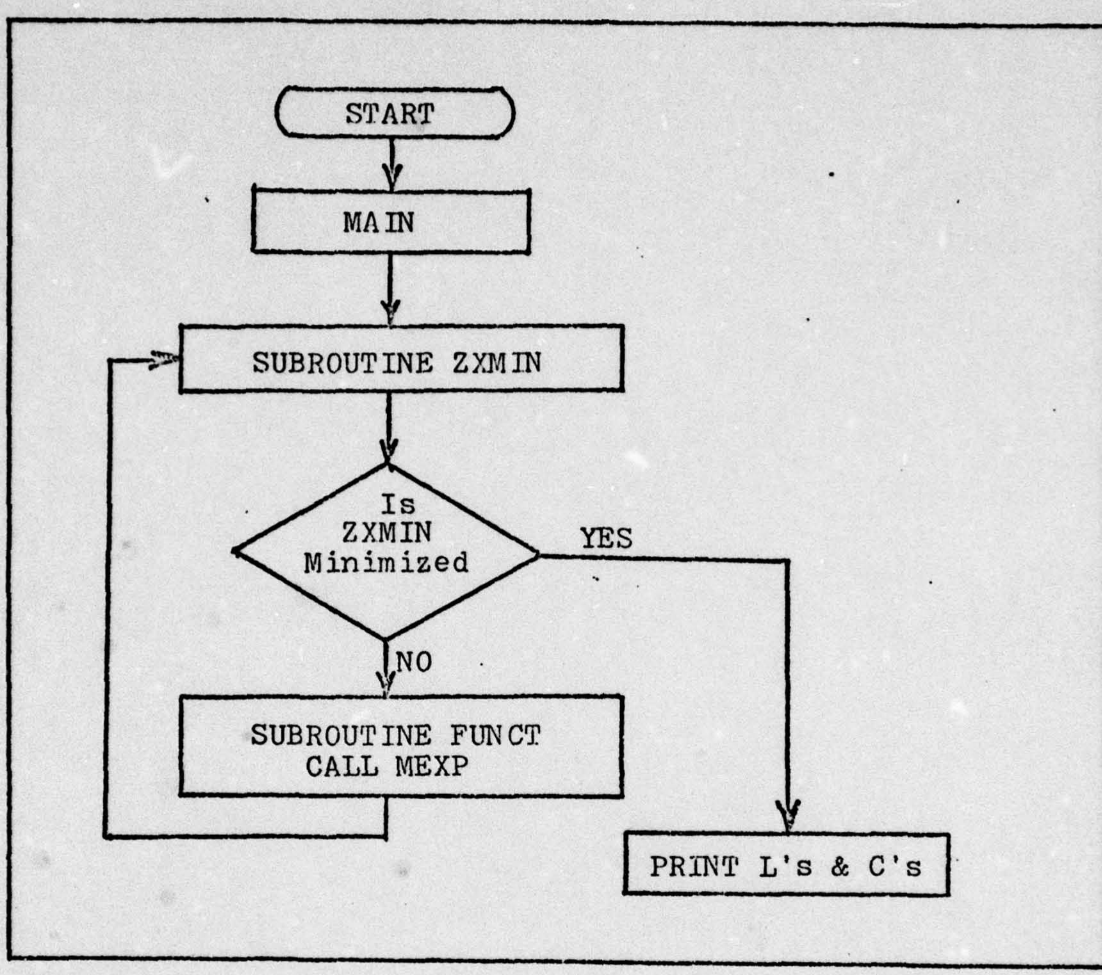

Figure 16. "Mini" Flow Chart of the Parameter Estimation Program

```
MON,T300,I070,CM60000,STCS8.T780499,VANNOY,53835
FTN,R=3.
ATTACH,IMSL,IMSL,ID=LIBRARY,SN=ASD.
ATTACH,PFN,CONTROL,CY=4,ID=L720033,SN=AFML.
LIBRARY,IMSL,PFN.
MAP,PART.
LGO.
```

These job cards are needed for the parameter estimator.

Parameter Estimation Source Listing

```
PROGRAM MIN          74/74    OPT=1                              FTN 4.6+446

          PROGRAM MIN(INPUT,OUTPUT)
          EXTERNAL FUNCT
C
C
C*********************************************************************
C*********************************************************************
C*********************************************************************
C*********************************************************************
C
C
C  THIS PROGRAM USES A LEAST SQUARES FIT OBJECTIVE FUNCTION MINIMIZING
C  SUBRIOTINE, ZXMIN, TO ESTIMATE THE INDUCTOR AND CAPICATOR VALUES
C  NEEDED FOR THE E-LINE TO PRODUCE A RECTANGULAR PULSE AT THE OUTPUT.
C  THE MAIN PROGRAM CONTAINS THE INITIAL GUESS FOR THE INDUCTOR AND
C  CAPACITOR VALUES.  THE MAIN PROGRAM ALSO CONTAINS PRINT STATEMENTS
C  AND VALUES FOR THE VARIABLES USED BY ZXMIN.
C
C
C*********************************************************************
C*********************************************************************
C*********************************************************************
C*********************************************************************
          DIMENSION X(10),H(55),G(10),W(30),C(5)
          REAL L(5)
C
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C        THE FOLLOWING VARIABLES ARE NEEDED BY ZXMIN (SEE THE OPERATING
C        INSTRUCTIONS FOR ZXMIN).
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C
          N = 10
          NSIG = 12
          MAXFN = 500
          IOPT = 0
C
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C        THESE VARIABLES ARE NEEDED BY THE MAIN PROGRAM.
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C
          NN = N/2
          NL = 0
          NM = 0
          IL = 0
          IM = 0
C
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C        THE INITIAL ESTIMATE OF THE INDUCTOR AND CAPACITOR VALUES WHICH
C        GIVE A RECTANGULAR OUTPUT VOLTAGE PULES ARE GIVEN BELOW:
C        (THE VALUES ARE IN HENRYS AND FARADS RESPECTIVELY)
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C
          L(1) = 2.577E-07
          L(2) = 2.086E-07
          L(3) = 2.171E-07
          L(4) = 2.554E-07
          L(5) = 3.607E-07
          C(1) = 7.830E-06
```

```
            C(2) = 7.782E-06
            C(3) = 8.521E-06
            C(4) = 1.079E-05
            C(5) = 2.029E-05
      C
      C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      C       SINCE THE PARAMETER VALUES USED BY THE SUBROUTINE ZXMIN MUST BE
      C       PUT INTO THE  X  VECTOR, THE INDUCTOR VALUES BECOME THE ODD
      C       ELEMENTS IN THE VECTOR AND THE CAPACITOR VALUES BECOME THE EVEN
      C       ELEMENTS IN THE VECTOR.
      C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      C
            DO 1 I=1,NN
            NL = 2*I-1
            NM = 2*I
            X(NL) = L(I)
            X(NM) = C(I)
          1 CONTINUE
      C
      C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      C       CALLING THE SUBROUTINE ZXMIN
      C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      C
            CALL ZXMIN(FUNCT,N,NSIG,MAXFN,IOPT,X,H,G,F,W,IER)
      C
      C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      C       PRINT STATEMENTS
      C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      C
            PRINT 5
          5 FORMAT(1H1)
            PRINT*,"   "
            PRINT*,"   "
            PRINT*,"   "
            PRINT*," THE INDUCTOR AND CAPACITOR VALUES AS CALCULATED BY "
            PRINT*," ZXMIN WHICH GIVE,AS CLOSE AS POSSIBLE, THE DESIRED"
            PRINT*," RECTANGULAR PULSE ARE:"
            PRINT*,"    "
            PRINT*,"    "
            PRINT*,"    "
      C
            DO 10 I=1,NN
            IL = 2*I-1
            IM = 2*I
            PRINT*," L(",I,") = ",X(IL),"            G(",IL,") = ",G(IL)
            PRINT*," C(",I,") = ",X(IM),"            G(",IM,") = ",G(IM)
         10 CONTINUE
      C
            PRINT*,"   "
            PRINT*,"   "
            PRINT*,"IER = ",IER
            PRINT*,"   "
            PRINT*,"   "
            PRINT*,"F = ",F
            PRINT*,"   "
            PRINT*,"   "
            DO 15 I=1,3
```



```
            PRINT*," W(",I,") = ",W(I)
         15 CONTINUE
            STOP
            END
```

```
C
C
C**********************************************************************
C**********************************************************************
C**********************************************************************
C
C
C  THIS SUBROUTINE TAKES THE INDUCTOR AND CAPACITOR VALUES DETERMINED
C  BY THE SUBROUTINE ZXMIN AND CALCULATES THE LEAST SQUARES OBJECTIVE
C  FUNCTION.  THE FUNCTION IS FOUND USING NUMERICAL INTEGRATION.
C  THROUGHOUT THE SUBROUTINE LIMITS HAVE BEEN SET ON CERTAIN VARIABLES
C  SO THAT OVERFLOW WILL NOT OCCUR.
C
C  THE COMMON STATEMENT AND THE VARIABLES NDIM, NDIM1, AND COM1 ARE
C  NEEDED BY THE SUBROUTINE WHICH CALCULATES THE STATE TRANSITION
C  MATRIX (MEXP).
C
C
C**********************************************************************
C**********************************************************************
C**********************************************************************
C
C
      SUBROUTINE FUNCT(N,X,F)
      DIMENSION X(N),A(11,11),EA(11,11),PAI(11,11),EAI(11,11),R(5)
      DIMENSION COM1(11,11)
      REAL J0,J1,J2,J3,JTOT,K1
      COMMON/MAIN1/NDIM,NDIM1,COM1
      NDIM = 11
      NDIM1 = 12
C
C
C**********************************************************************
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C
C       THE FOLLOWING IS A LIST OF THE VARIABLES NEEDED IN THE LEAST
C       SQUARES FIT OBJECTIVE FUNCTION SUBROUTINE.
C       N1------THE DIMENSION OF THE A MATRIX
C
C       DELT----THE TIME SUBINTERVAL USED IN THE NUMERICAL INTEGRATION
C
C       X0------THE INITIAL VOLTAGE ON THE SOURCE CAPACITOR
C
C       TP------TIME WHEN THE DESIRED PULSE GOES TO ZERO (SEE EQUASION
C               35 IN THIS TEXT).
C
C       PULHT---THE DESIRED PULSE HEIGHT
C
C       IPULSE--TIME WHEN THE DESIRED PULSE GOES TO ZERO,(TP), DIVIDED
C               BY DELT.  THIS INTEGER IS AN UPPER LIMIT ON ONE OF THE
C               SUMMERS IN THE OBJECTIVE FUNCTION (SEE EQASION 35 IN
C               THIS THESIS).
C
C       ITOT----TIME WHEN THE OSCILLATIONS OF THE ACTUAL PULSE SHOULD
C               STOP, (3TP), DIVIDED BY DELT.  THIS INTEGER IS THE UPPER
C               LIMIT ON THE SUMMER IN THE OBJECTIVE FUNCTION.
C
```

```
C         I-------USED IN THE INTEGRATION ROUTINE  =1,2,3,...,3TP/DELT.
C
C         Y-------THE SOLUTION TO THE OUTPUT EQUASION.  WHEN Y IS USED
C                 IN THE INTEGRATION ROUTINE ITS VALUE IS NEEDED AT
C                 TIME  T=I*DELT  WHERE  I=1,2,3,...,3TP/DELT.
C
C         K1------THE WEIGHTING FACTOR
C
C         CS------THE CAPACITANCE OF THE SOURCE CAPICATOR IN FARADS.
C
C         R(J)----THE VALUE OF THE RESISTOR IN THE J'TH MESH.
C                 J=1,2,3,...,M;  WHERE M IS THE NUMBER OF MESHES.
C
C         RL------THE RESISTANCE OF THE LOAD RESISTOR IN OHMS.
C
C         F-------THE CALCULATED VALUE OF THE OBJECTIVE FUNCTION FOR THE
C                 INDUCTANCES AND CAPACITANCES ESTIMATED BY ZXMIN.
C
C         A-------THE  MXM  PLANT COEFFICIENT MATRIX  (SEE EQUASION 15 IN
C                 THIS THESIS).  THE MATRIX IS INITIALLY SET TO ZERO AND
C                 THEN ALL NON-ZERO ILEMENTS ARE DEFINED.
C
C
C         THE FUNCTION SUBROUTINE USES SEVERAL OTHER SUBROUTINES THEY ARE:
C
C         MEXP----A SUBROUTINE THAT CALCULATES THE STATE TRANSITION
C                 MATRIX.
C
C         MMPY----A SUBROUTINE THAT COMPUTES THE MATRIX PRODUCT C=A*B.
C
C         MZERO---A SUBROUTINE THAT ZEROS A MATRIX, A=0.0.
C
C         MIDENT--A SUBROUTINE THAT CREATES THE IDENTITY MATRIX.
C
C         MEQUAL--A SUBROUTINE THAT MAKES ONE SUBROUTINE EQUAL ANOTHER, B=A
C
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C*********************************************************************
C
C
C
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C         THE INITIAL VALUES FOR THE VARIABLES ARE GIVEN HERE.
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C
      N1 = 11
      CALL MZERO(A,N1,N1)
      CALL MZERO(EA,N1,N1)
      CALL MZERO(PAI,N1,N1)
      CALL MZERO(EAI,N1,N1)
      DELT = 1.0E-06
      X0 = 30.0E+03
      Y = 0.0
      PULHT = 30.0E+03
      JTOT = 0.0
      IPULSE = 25
      ITOT = 65
```

```
      J0 = 0.0
      J1 = 0.0
      J2 = 0.0
      J3 = 0.0
      K1 = 0.25
      F = 0.0
      CS = 1.333E-04
      R(1) = 15.0E-03
      R(2) = 15.0E-03
      R(3) = 15.0E-03
      R(4) = 15.0E-03
      R(5) = 15.0E-03
      RL = 0.165
C
C * * * * * * * * * * * * * * * * * * * * * * * *
C      ALL NON-ZERO ELEMENTS IF THE A MATRIX ARE DEFINED HERE.
C * * * * * * * * * * * * * * * * * * * * * * * *
C
      A(1,2) = -1.0/CS
      A(2,1) = 1.0/X(1)
      A(2,2) = -R(1)/X(1)
      A(2,3) = -1.0/X(1)
      A(3,2) = 1.0/X(2)
      A(3,4)= -1.0/X(2)
      A(4,3) = 1.0/X(3)
      A(4,4) = -R(2)/X(3)
      A(4,5) = -1.0/X(3)
      A(5,4) = 1.0/X(4)
      A(5,6) = -1.0/X(4)
      A(6,5) = 1.0/X(5)
      A(6,6) = -R(3)/X(5)
      A(6,7) = -1.0/X(5)
      A(7,6) = 1.0/X(6)
      A(7,8) = -1.0/X(6)
      A(8,7) = 1.0/X(7)
      A(8,8) = -R(4)/X(7)
      A(8,9) = -1.0/X(7)
      A(9,8) = 1.0/X(8)
      A(9,10) = -1.0/X(8)
      A(10,9) = 1.0/X(9)
      A(10,10) = -R(5)/X(9)
      A(10,11) = -1.0/X(9)
      A(11,10) = 1.0/X(10)
      A(11,11) = -1.0/(RL*X(10))
C
C * * * * * * * * * * * * * * * * * * * * * * * *
C      LIMITS ARE SET HERE ON HOW LARGE THE ELEMENTS IN THE A MATRIX
C      CAN BE.
C * * * * * * * * * * * * * * * * * * * * * * * *
C
      EAMAX = 610.0/DELT
      DO 10 I=1,N1
      DO 10 J=1,N1
      IF(A(I,J).GT.EAMAX) GO TO 11
      GO TO 10
   11 A(I,J) = EAMAX
   10 CONTINUE
```

```
      C
      C*******************************************************************
      C   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
      C
      C
      C
      C        THIS SECTION FINDS THE FUNCTION F FOR THE ESTIMATED VALUES
      C        OF THE L'S AND C'S.  NUMERICAL INTEGRATION IS USED.
      C
      C
      C   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
      C*******************************************************************
      C
      C
      C   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
      C        THE SUBROUTINE MEXP IS CALLED HERE, IT CALCULATES THE STATE
      C        TRANSITION MATRIX AT TIME  T=DELT.  LIMITS ARE ALSO SET ON HOW
      C        LARGE THE MATRIX CAN BE.  THE VARIABLES USED BY MEXP ARE DEFINED
      C        IN THE OPERATING INSTRUCTIONS FOR MEXP.
      C   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
      C
            CALL MEXP(N1,A,DELT,EA)
      C
            DO 5 I=1,N1
            DO 5 J=1,N1
            IF(EA(I,J).GT.1.0E+50) GO TO 6
            IF(EA(I,J).LT.-1.0E+50) GO TO 7
            GO TO 5
          6 EA(I,J) = 1.0E+50
            GO TO 5
          7 EA(I,J) = -1.0E+50
          5 CONTINUE
      C
      C   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
      C        Y IS THE SOLUTION TO THE OUTPUT EQUASION AT THE PARITCULAR
      C        TIME T=I*DELT.
      C        LIMITS ARE ALSO SET HERE ON HOW LARGE Y CAN BE.
      C   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
      C
            CALL MIDENT(PAI,N1,N1)
      C
            DO 100 I=6,ITOT
            CALL MZERO(EAI,N1,N1)
      C
            CALL MMPY(EA,PAI,EAI,N1,N1,N1)
      C
            CALL MEQUAL(PAI,EAI,N1,N1)
      C
            Y = EAI(N1,1)*X0
      C
            IF(Y.GT.1.0E+50) GO TO 12
            IF(Y.LT.-1.0E+50) GO TO 13
            GO TO 14
         12 Y = -1.0E+50
            PRINT*,"Y IS LESS THAN -1.0E+50"
            GO TO 14
         13 Y = 1.0E+50
```

```
      PRINT*,"Y IS GREATER THAN 1.0E+50"
   14 CONTINUE
      IF(I.EQ.1) GO TO 15
      IF(I.EQ.ITOT) GO TO 20
      IF(I.GT.IPULSE) GO TO 25
      PULJ = ((PULHT-Y)**2)*DELT
      J1 = J1 + PULJ
      PULJ = 0.0
      GO TO 30
   15 CONTINUE
      J0 = ((PULHT-Y)**2)*DELT/2.0
      GO TO 30
   25 CONTINUE
      ZEROJ = K1*(Y**2)*DELT
      J2 = J2 + ZEROJ
      ZEROJ = 0.0
      GO TO 30
   20 CONTINUE
      J3 = K1*(Y**2)*DELT/2.0
   30 CONTINUE
      Y = 0.0
  100 CONTINUE
      F = J0 + J1 + J2 + J3
      PRINT*," F = ",F
      RETURN
      END
```

```
      SUBROUTINE MMPY(A,B,C,M,K,N)
C
C
C***********************************************************************
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C
C
C    THIS SUBROUTINE WILL COMPUTE THE MATRIX PRODUCT C=A*B
C
C
C    USAGE:
C    DIMENSION A(M,K),B(K,N),C(M,N)
C    CALL MMPY(A,B,C,M,K,N)
C
C
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C***********************************************************************
C
C
      DIMENSION A(M,K),B(K,N),C(M,N)
      DO 10 J=1,N
      DO 10 I=1,M
      C(I,J) = 0.0
      DO 10 L=1,K
      C(I,J) = C(I,J) + A(I,L)*B(L,J)
   10 CONTINUE
C
C
      DO 20 I=1,M
      DO 20 J=1,N
      IF(C(I,J).GT.1.0E+50) GO TO 35
      IF(C(I,J).LT.-1.0E+50) GO TO 30
      GO TO 20
   30 C(I,J) = -1.0E+50
      GO TO 20
   35 C(I,J) = 1.0E+50
   20 CONTINUE
C
C
      RETURN
      END
```

```
      SUBROUTINE MZERO(A,M,N)
C
C
C***********************************************************************
C  * * * * * * * * * * * * * * * * * * * * * * * *
C
C
C    THIS SUBROUTINE ZEROIS A MATRIX A=0.0
C
C    USAGE:
C    DIMENSION A(M,N)
C    CALL MZERO(A,M,N)
C
C
C  * * * * * * * * * * * * * * * * * * * * * * * *
C***********************************************************************
C
C
      DIMENSION A(M,N)
C
      DO 10 I=1,M
      DO 10 J=1,N
      A(I,J) = 0.0
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MIDENT(D,M,N)
C
C
C***********************************************************************
C  *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
C
C
C   THIS SUBROUTINE CREATES THE IDENTITY MATRIX
C
C   USAGE:
C   DIMENSION D(M,N)
C   CALL MIDENT(D,M,N)
C
C
C  *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
C***********************************************************************
C
C
      DIMENSION D(M,N)
C
      IF(M.NE.N) GO TO 20
C
      CALL MZERO(D,M,N)
C
      DO 10 I=1,M
      D(I,I) = 1.0
   10 CONTINUE
      GO TO 30
C
   20 PRINT*,"ERROR: MATRIX IS NOT SQUARE:CAN'T FORM THE IDENTITY MAT"
C
   30 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MEQUAL(A,B,M,N)
C
C
C*************************************************************************
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C
C
C   THIS SUBROUTINE MAKES ONE MATRIX EQUAL TO ANOTHER MATRIX
C
C   USAGE:
C   DIMENSION A(M,N),B(M,N)
C   CALL MEQUAL(A,B,M,N)
C
C
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C*************************************************************************
C
C
      DIMENSION A(M,N),B(M,N)
C
      DO 10 I=1,M
      DO 10 J=1,N
      A(I,J) = B(I,J)
   10 CONTINUE
      RETURN
      END
```

Typical Output from the Estimation Program

```
THE INDUCTOR AND CAPACITOR VALUES AS CALCULATED BY
ZXMIN WHICH GIVE,AS CLOSE AS POSSIBLE, THE DESIRED
RECTANGULAR PULSE ARE:


L(1) = 2.150479715151E-7            G(1) = -727714059.8737
C(1) = .000007826058300259            G(2) = 43130141.9701
L(2) = 5.244114161551E-8            G(3) = 50554195.62787
C(2) = .000037779038613154            G(4) = 35561728.86354
L(3) = 1.516190359001E-7            G(5) = 997279413.9773
C(3) = .000003514937499577            G(6) = 29631418.78898
L(4) = 1.639374973 69E-9           G(7) = -4.097059552653E+102
C(4) = .000010791866031904           G(8) = 30523405.95086
L(5) = 1.270040401654E-7            G(9) = 1.644976441214E+9
C(5) = .000020286712508 29            G(10) = 16692788.39131


IER = 130


F = 7655.104586558


W(1) = 4.097059552653E+102
W(2) = 418.
W(3) = -103.1792786124
```

Vita

Larry W. Vannoy was born on 20 December 1954 in Blackwell, Oklahoma. He graduated from Hillcrest High School in Dalzell, South Carolina in 1973. He then attended the University of South Carolina where he received, in 1977, the Bachelor of Science Degree with concentration in Electrical Engineering. Upon graduation he received a commission in the United States Air Force. In September he was directed to the School of Engineering, Air Force Institute of Technology, as his initial active duty assignment.

Permanent Address: 5413 Plantation Drive
Sumter, South Carolina 29150

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Pulse-Forming Networks
Network Simulation
Digital Computer Simulation
Parameter Estimator
Line Pulser

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A line-type, voltage-fed, pulse-forming network, called the "E-line," is studied. The energy to be delivered to the load is stored in a source capacitor at the front of the network. Storing all the energy in the source capacitor leads to a more reliable and lightweight system than is achievable by a conventional network where all the capacitors store energy. The network is to deliver 100-200 rectangular pulses per second to

next page

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Block 20 continued.

the load where each pulse is 20-30 microseconds in duration and has a pulse height of 30,000 volts.

A computer program is developed which models the discharge of the network and is used to analyze the output pulse shape for different inductor and capacitor values within the network. Another computer program is developed which estimates the inductor and capacitor values needed for the network to give a close approximation to the desired rectangular pulse. The simulation program models the discharge of the actual network very well. The estimation program as applied in this thesis does a poor job of predicting the inductor and capacitor values needed. The desired rectangular pulse is not achieved. Recommendations are included which may improve the simulation program, estimation program, and the actual pulse shape.